

Airline Revenue Management: Sell-up and Forecasting Algorithms

by

Thomas O. Gorin

Ingénieur des Arts et Manufacture (ECP 1999)

Submitted to the Department of Aeronautics and Astronautics and Center for
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Master of Science in Transportation

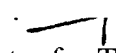
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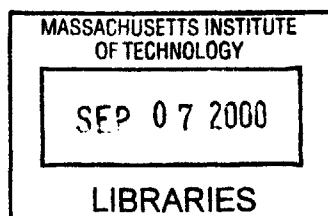

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ABSTRACT

Recent technological improvements have allowed airlines to implement sophisticated Revenue Management systems in order to maximize revenues. Computational capabilities make it possible to perform network-based analysis of supply and demand and therefore to increase the gains achieved with the help of “O-D control” Revenue Management algorithms. However, the more commonly used and cheaper flight leg-based algorithms have not yet been used to the best of their potential and can still benefit from better modeling of passenger behavior.

Our first purpose in this thesis is therefore to evaluate the benefits of incorporating sell-up models into current leg-based airline Revenue Management algorithms. Another question we would like to try and address is whether it would be possible to improve the leg-based models to reach revenue gains comparable to those of O-D control algorithms. To try and achieve this goal, we improve the modeling in our leg-based Revenue Management algorithms by accounting for the possibility of sell-up, that is the probability that a passenger will accept a more expensive ticket than originally desired if seats are not available at the lower fare. In addition, previous research has shown that there are revenue gains to be achieved through better forecasting, therefore, we also evaluate the use of better forecasting methods and quantify their revenue impact. In particular, we focus our efforts on understanding the impact of the unconstraining models on revenue gains by using various detraction methods and comparing their effect.

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INTRODUCTION

In this thesis, our goal is to evaluate the revenue benefits that can be achieved by an airline if it improves its models of passenger behavior. In the recent past, airlines have been looking for ways to increase their revenues. In particular, airlines started segmenting demand through differential pricing, that is, recognizing the fact that there are different types of passengers who respond differently to prices, availability, level of service, etc. The next step was then to develop Revenue Management algorithms that allowed airlines to increase total passenger revenues by protecting seats for late-booking high fare passengers on high demand flights, while making more seats available to early-booking low fare passengers on flights expected to have empty seats. Our goal will be to simulate the possible revenue gains that can be achieved through better understanding and modeling of passenger behavior, and point out the potential for yet unachieved revenue gains.

To do this, we will be using the Passenger Origin Destination Simulator (PODS) developed at the Boeing Company by Hopperstad to model an airline in a competitive setting with various levels of capabilities in terms of the finesse of the model of a real life environment. In this context, we will try to reach our previously described objective.

In the following paragraphs, we will introduce the fundamental concepts of Revenue Management, demand forecasting and sell-up. Sell-up will be of particular interest to us in this thesis, as it is a passenger behavior that has not yet been combined with Revenue Management algorithms, even though sell-up models have already been developed.

Revenue Management

Revenue Management, also known as Yield Management, has been the subject of much research in the past few decades. It represents the determination of the airlines to increase their revenues as much as possible. The fundamental idea behind Yield Management lies in the term “yield”. In the airline industry, “yield” refers to the dollar amount paid by a passenger on a per-mile or per-kilometer basis. Hence, when managing yield, the airlines clearly try to attract as many high yield passengers as possible, rather than high revenue passengers (even though the two are not incompatible). In this section, we will give a very brief overview of the history of the

airline industry and the advent of Revenue Management. In particular, we will discuss the changes that were brought about by deregulation.

Before deregulation, airlines would typically not set their own fares; they would be determined on a per-mile basis by the Civil Aeronautics Board (CAB), and independently of the origin-destination (O-D) market. If the airlines were to suffer losses, the Civil Aeronautics Board would allow fare increases to compensate for the airline's losses. Similarly, the airlines did not have to worry about costs, be they maintenance or labor related, as the CAB would again allow fare increases to cover the operating costs. Within this context, airlines then offered an extremely high quality service to passengers, with a high level of in-flight services and good frequencies of flights. However, this high level of service had inevitable downsides, such as low load factors.

In 1978, after US airline deregulation, the airlines were faced with the problem of increasing their revenues and decreasing their costs in the new competitive environment. Competition naturally led to O-D fares that strongly depended on the market and its attractiveness to passengers, and less on the distance flown. In the process of increasing their revenues, the airlines found out that the most efficient way to achieve higher profits was to charge passengers their maximum willingness-to-pay (WTP). However, this economic theory cannot be fully realized in any industry since it would mean charging each customer a different price for the same service or goods. Therefore, Revenue Management was developed to try and approach optimality in terms of prices charged to passengers. Before deregulation, airlines had attempted to introduce differential pricing in order to attract more passengers on their empty flights, but on a very small scale.

Yield Management led to extremely complicated fare structures as the airlines were trying to get as many high yield passengers as possible, and yet to fill up their airplanes at the same time. To ensure these two goals, they introduced fences, also known as restrictions, that prevented business passengers from meeting the requirements to buy low fare tickets and yet allowed leisure passengers, with more flexible schedules, to buy these tickets. This was the first step to filling up the airplanes without losing revenue from business passengers. Examples of fences are:

- Advance purchase
- Saturday night stay
- Round trip purchase requirement
- Non refundability or partial refundability

These fences serve their purpose well, as it is clear that a business passenger will in most cases not know his schedule well in advance or be willing to stay over a Saturday night. Similarly, the round trip requirement prevents passengers from traveling to several cities consecutively.

Yet, to increase revenues, airlines had to get as many high yield business passengers as possible on board the planes. To do this, they needed forecasts of the demand for future flights, and then would assign a certain number of seats to a certain fare category. This allocation of seats to a specific fare class with its associated restrictions is the essence of what is commonly referred to as Revenue Management.

Yield Management has today become much more complex. The goal has now become to optimize the seat allocation process, and this has led to new forecasting techniques, as well as a number of seat allocation algorithms. There are two types of seat allocation algorithms: flight-leg based, Fare Class algorithms, such as Expected Marginal Seat Revenue (EMSR) algorithms (EMSRa or EMSRb), and O-D algorithms, such as Displacement Adjusted Virtual Nesting (DAVN) or Network Bid Price (Netbid). By Origin-Destination (O-D) algorithm, we mean any type of algorithm that either protects seats or forecasts demand on a per O-D market basis as opposed to a per leg basis. In airline terms, a market is a pair of origin and destination cities, such as Boston-Miami for example, regardless of the number of flight legs required to fly from the origin city to the destination city.

All these Revenue Management algorithms are aimed at maximizing the airlines' revenues. To achieve this goal, they use mathematical formulas to forecast demand and protect seats on a flight according to the forecasts. Hence, there are two major issues in Revenue Management: demand forecasting and seat protection. Seat allocation algorithms are numerous, as discussed earlier. Demand forecasting currently implies using techniques based on past historical data. However, demand modeling also requires modeling passenger behavior, such as "willingness-to-pay", possibilities to meet various requirements and fences, and sell-up behavior, which is the major focus of this thesis.

In the following paragraphs, we will come back to forecasting and sell-up in more detail so as to give a definition and a brief description of the mechanisms involved.

Forecasting

To try and predict future demand, airlines use forecasting techniques. These are numerous and rely on historical data. For example, to forecast demand for a Friday evening flight, airlines will use data from the previous weeks' Friday evening flights and estimate what the actual demand will be for this particular Friday evening flight. However, given the many parameters in play, it is very difficult to obtain an accurate forecast. Indeed, demand varies with economic conditions, seasons, days of the week and time of day. Hence, it is very tricky to determine what portion of the historical data should be used to forecast demand.

Moreover, there also is the problem of previous flights that sold out, and that therefore cannot be used directly as raw data. In order to be able to use these full flights in our forecasting algorithms, we need to apply a transformation that will let us estimate the actual demand for these flights. Indeed, the capacity on a flight

clearly limits the number of bookings and hence does not reflect actual demand when the flight is sold out. This process of unconstraining demand is also known as detruncation. These methods are forecasting methods that predict what the actual data would have looked like, had there been no capacity constraints.

Therefore, forecasting is basically divided into two major parts: detruncation and actual forecasting. Once again, detruncation consists of unconstraining since a flight may sell out, or close down, for capacity reasons. It then becomes very difficult to predict what the actual demand, the unconstrained demand, was for the flight. This is the role of the detruncation method: it projects a closed observation to an estimate of the unconstrained demand for the associated flight. Forecasting then uses this data to project what the future demand for a given flight will be. The major difference between forecasting and detruncation lies in the fact the forecasting predicts future demand, whereas detruncation “predicts” or rather estimates past demand.

Overall, demand forecasting has an enormous impact on Revenue Management algorithms. Several forecasting and detruncation algorithms are available for our use, as we will discuss later in this thesis.

Sell-up

In any origin-destination market, a given discretionary passenger will seek the lowest available price. In the event that a passenger cannot book his desired flight and fare class, he then may choose one of the following options:

- Horizontally shift to the same airline, but on a different flight, and in the same fare class.
- Spill to another airline.
- Not travel at all.
- Vertically shift to a higher fare class on the same flight and airline.

(Source: Belobaba, Peter P., Air Travel Demand and Airline Seat Inventory Management, 1987)

The last possibility is what is referred to as sell-up, and constitutes an important part of current research. In particular, as stated earlier, we will study this passenger behavior in order to assess the revenue gains that can be achieved without necessarily improving the network or Revenue Management models used by the airlines.

Goal of the Thesis

Currently, most airlines use leg-based algorithms to manage their seat inventory. Only the more advanced airlines use O-D control algorithms that forecast demand and protect their seating capacity or inventory at the network level. In this environment, other airlines are considering switching to a more advanced network Revenue Management method, in order to increase their revenues and therefore their profits.

The question that we raise here is whether a first step to higher revenues would not be to upgrade the existing leg-based, Fare Class Revenue Management algorithms in order to achieve revenues similar to those of O-D algorithms. Indeed, the financial implications of upgrading the entire Revenue Management system are enormous. Hence, the goal of the thesis is first of all to assess the revenue benefits of better modeling passenger behavior and second to establish whether or not it would be possible to increase the airline's revenues to make them comparable to those of the better O-D methods.

Specifically, airlines are currently interested in the implications of accounting for sell-up and improving the detruncation and forecasting methods. Therefore, in this thesis we will incorporate the notion of sell-up in the typical leg-based, Fare Class Revenue Management EMSRb algorithm, and improve the detruncation and forecasting methods in order to increase, hopefully, the revenues of the airline using this leg-based algorithm. We will evaluate the impact of these changes for both the airline leading the change and its competitor. Moreover, we will test the robustness of the results achieved by the airline accounting for sell-up by changing the Revenue Management method of the competitor.

Previous studies have shown that O-D algorithms can bring as much as 2.5 percent revenue gains over the basic leg-based EMSRb algorithm. We will therefore focus on the benefits of accounting for sell-up in a leg-based Revenue Management environment such as EMSRb, and try and reach revenue gains similar to or better than those of the O-D algorithms by accounting for sell-up, if it is possible.

To do this, we will be using the Passenger Origin Destination Simulator (PODS), developed by Hopperstad from the Boeing Company and to be described in detail in Chapter 1.1. This software enables us to run simulations of a network of routes and airlines competing on the various markets and gives us the revenues for all the airlines. We will be running a number of simulations to answer our previous questions regarding sell-up. The simulator enables us to input the sell-up rates, choose from a variety of forecasting methods, and decide what type of Revenue Management methods (O-D or leg-based) the competing airlines will be using.

Structure of the Thesis

The thesis will be divided into two major parts. In the first part, we will introduce and explain the fundamental concepts that are used in the thesis. In particular, in the first chapter, we will explain how the Passenger Origin Destination Simulator (PODS) works. We will get into the details of the structure of the simulator and briefly describe the mechanisms of the algorithms that are used. We will then discuss Revenue Management more thoroughly and get into the details of the algorithms used in PODS. In particular, we will describe six of the more commonly used algorithms in the airline industry today, divided into leg-based algorithms and O-D based algorithms. These six algorithms that we focus on are Expected Marginal Seat Revenue (EMSR), Greedy Virtual Nesting (GVN), Displacement Adjusted Virtual Nesting (DAVN), Heuristic Bid Price (HBP), Network Bid Price (Netbid) and Prorated Bid Price (ProBP).

In the second chapter, we then discuss forecasting and detruncation methods. We first discuss detruncation methods and focus on the ones more commonly used in the airline industry, namely Booking Curve detruncation (BC), Projection detruncation (Proj) and Adjusted Booking Curve detruncation (AdjBC). The latter was recently described and tested by **Bratu**¹ to account for low biases in unconstraining mechanisms, as an improvement of the more typical BC. Second, we focus on forecasting methods, Regression forecasting (REG) and Pick-up forecasting (PU).

In the third and final chapter of this first part, we come back to the notion of sell-up and explain in more details the phenomena that are responsible for this behavior of passengers. In particular, we will discuss passenger segmentation, price discrimination and maximum willingness-to-pay (WTP). We will also explain how this sell-up potential can be incorporated in several Revenue Management algorithms with the **Belobaba-Weatherford**² heuristic.

By the end of this first part, we hope to have explained enough about Revenue Management and sell-up behaviors for the reader to understand the results presented in the following chapters and the relevance of the topic to airlines.

In the second part, we focus on simulations and results pertaining to sell-up and better detruncation. Again, this second part will be divided into several chapters. Chapter 4 is the first chapter of this second part, focusing on a leg-based, Fare Class Revenue Management environment. More specifically, we study what happens when airline A accounts for sell-up and upgrades to more aggressive detruncation methods (i.e. methods that lead to higher estimates of the unconstrained demand) as a function of what the competitor does.

¹ Bratu, Stéphane J-C., *Modified Booking Curve Detruncation and Sell-up Techniques*, PODS Summit IX Presentation

² Belobaba, P.P., Weatherford, L.R., *Comparing Decision Rules that Incorporate Customer Diversion in Perishable Asset Revenue Management Situations*, Decision Sciences, Volume 27, Spring 1996

In Chapter Five, we study what happens to an airline that accounts for sell-up in an O-D based environment, as opposed to a leg-based environment. In this paragraph we simulate essentially the same cases as in our previous analysis, but airline A now uses GVN combined with sell-up.

Finally, in Chapter Six, we compare the results we achieved with previously observed results from analyses of the Revenue Management methods available to airlines. In this chapter we also try and validate our results in a bigger network, but more importantly study the effects of this larger network on the gains observed previously. Finally, we use the capabilities of PODS to compare the sell-up rates input in the simulations (i.e. the estimated sell-up rates on our part) to the actual observed sell-up rates at the end of the simulation.

In conclusion, we will have, if not proven, at least strongly suggested that both accounting for sell-up and improving the detruncation methods can lead to higher revenues for the airline implementing this minor change in its Revenue Management system. Moreover, the order of magnitude of the gain is the same as the gains obtained by some O-D Revenue Management methods.

PART I: THEORY

In this first part of the thesis, we focus on a few critical details that must be understood before we can present our results. In particular, we define the important concepts of Revenue Management, Forecasting, Detruncation and Sell-up. We also describe the simulator we used to obtain our results.

This first part is therefore divided into three chapters. The first chapter deals with the introduction of the simulator and the general task of explaining what Revenue Management is and why it is very important to the airlines today. The second chapter gets into the details of Forecasting and Detruncation methods, as they will play an important role in our simulations. Finally, the third and last chapter of this first part deals with the notion of Sell-up. We explain what Sell-up is and how previous research has proposed to account for it in a Revenue Management environment.

Chapter 1: Pods And Revenue Management

In this chapter, we will first introduce the Passenger Origin Destination Simulator that we have been alluding to in the introduction. We will get into the basics of how it works and how it simulates a realistic airline environment. In a second part, we will focus on Revenue Management algorithms, be they Fare Class or O-D based. We will point out the specificity of each algorithm and provide a general understanding of each of them. Overall, this chapter will be devoted to the two fundamental elements in this research study, namely the simulator and the critical notion of Revenue Management in the airline industry.

1.1 PODS

For the purpose of this thesis, we used a simulator, the Passenger Origin Destination Simulator (PODS), developed at Boeing by Hopperstad³. This simulator evolved from the Decision Window Model⁴ (DWM), also developed at Boeing. However, PODS incorporates major additional simulation capabilities, in that in addition to the DWM characteristics, PODS also takes into consideration the fares offered on each flight and path, as well as the restrictions associated with each fare category on a specific flight. Originally, the DWM choice model determined passenger preferences based on the schedule offered by the airlines, the airline image and a set of other factors, such as the aircraft type. PODS uses the same logic as the DWM model, includes fares and associated restrictions, and also allows the two (or more) competing airlines to simulate the effects of various Revenue Management methods on their network revenues.

The greatest improvement from previous Revenue Management simulators, such as MITSIM (c.f. **Williamson**⁵, **Mak**⁶), for example, is that PODS allows the simulation of a competitive environment where passengers actually choose among paths and airlines. Moreover, the demands are correlated across passenger types and markets, unlike previous simulations. As explained by **Mak**⁶, MITSIM requires the user to input the mean demand for each fare class. In addition, the forecaster evaluates future demand by summing the mean incremental ODF demands for all remaining periods. That is, the forecaster uses mean incremental input demands, while PODS uses historical data. This is the major difference with PODS, as demand is generated by the passengers' choice in the simulator. In addition, there are two competing airlines, which was not the case with MITSIM.

³ Boeing PODS, developed by Hopperstad, Berge and Filipowski

⁴ *Decision Window Path Preference Model (DWM)*, The Boeing Company, February 1994

⁵ Williamson, Elisabeth L., *Airline Network Seat Inventory Control: Methodologies and Revenue Impacts*, June 1992

⁶ Mak, Chung Y., *Revenue Impacts of Airline Yield Management*, January 1992

PODS is able to simulate an entire network of origin-destination markets for one or more airlines and enables us to then use its outputs to analyze the competitive implications of various Revenue Management methods on airline revenues. Moreover, PODS allows the user to simulate competing airlines with different Revenue Management and forecasting methods. The fundamental characteristic of PODS is that it simulates the booking process for a single day's departures on a network, with **competing** airlines using different fare structures and (or) different Revenue Management methods.

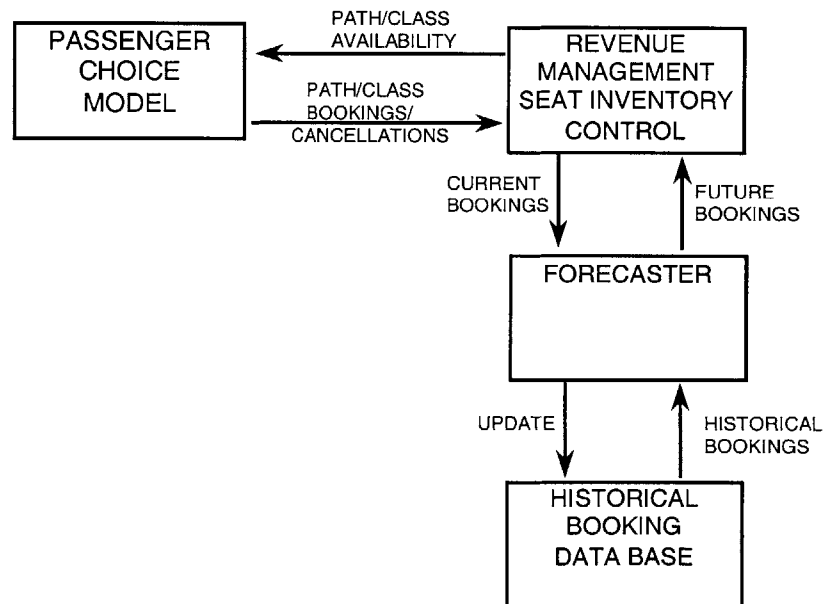
In the following paragraphs, we will briefly describe the model, its inputs and its outputs.

1.1.1 The model

Our primary use of PODS is to simulate a competitive environment in which various airlines operate a pre-determined network of flights. Given this situation, we then try to analyze the influence of Revenue Management on different measures such as airline network revenues or load factors.

Conceptually, PODS is composed of four interrelated elements: the historical booking database, the forecaster, the Revenue Management optimizer and the passenger decision model. These four components are linked as shown on Figure 1.

Figure 1: PODS Architecture



Source: Hopperstad, The Boeing Company

The historical booking database collects information from previous flights and feeds this historical data into the forecaster, whose historical database is manually initialized at the beginning of each simulation run. The forecaster then uses this data,

along with bookings currently on hand, provided by the Revenue Management optimizer, to forecast future demand for a given flight. These expected future bookings are then fed into the Revenue Management optimizer. With this data and actual path and class bookings and cancellations, the Revenue Management optimizer then determines seat protections and availability. Finally, this data is fed into the passenger choice model, which uses it to assign new prospective passengers to available path-fare combinations according to their decision window and budget. All this information is then input back into the Revenue Management optimizer as historical data to be used by the airline for future flight departures.

On a finer level, the simulations can be described as a set of trials. Each simulation, or case, has specific input parameters that determine the type of forecaster and Revenue Management optimizer that will be used by each airline, along with the network definition. In the following paragraphs, we describe the structure of PODS in a little more detail, but the reader is referred to **Wilson**⁷ and **Lee**⁸ for more exhaustive explanations.

Each case was chosen to be subdivided into 20 trials, each trial being composed of 600 samples, 200 of which are discarded in order to avoid initial conditions effects. This amount of trials and samples was chosen in order to get statistically significant results, as explained by **Lee**⁸. The first 200 samples are discarded in order to eliminate the initial conditions' effects, while the 20 trials provide stable results. Each sample contains one set of flight departures for the network representing a single day of operations.

Overall, we have a set of 12,000 samples per case, 8000 of which provide us with actual data gathered in the output files. These samples are separated among 20 trials in order to reduce the correlation between the samples. Indeed, every sample is influenced by the conditions in which the system is when the sample is run. This condition is a result of the previous sample run, or the previous departure day bookings. Therefore, to reduce the correlation, the model is split in 20 trials of 600 samples each. This high number of 600 samples was chosen in order to obtain steady-state results that actually give us reliable information. Moreover, as we discussed earlier, the first 200 samples are discarded in order to eliminate the initial conditions' effect. It has been determined in previous research work by **Lee**⁸ that 600 samples with 200 burns yields statistically significant and stable results while at the same time uses reasonable computational power.

The demand generation processes are then created using DWM according to the following steps. First, the system generates a window that is set according to the passenger's earliest possible departure time and latest possible arrival time. Second, once this window has been determined, the model generates the paths that are compatible with this window for each given passenger. Finally, given these paths, the first choice path is systematically generated, according to airline image, path

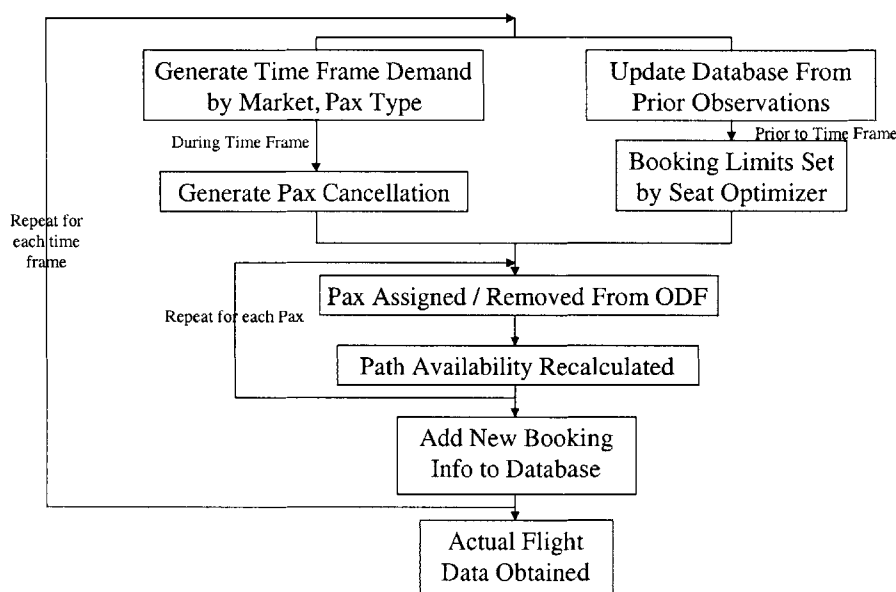
⁷ Wilson, John L., *The Value of Revenue Management Innovation in a Competitive Airline Industry*, May 1995

⁸ Lee, Alex Y., *Investigation of Competitive Impacts of Origin-Destination Control Using PODS*, June 1998, Chapter 3.1.2, pp43-44

quality (i.e. number of connections, etc.) and other factors. Once the paths are generated, we then get time of day distribution for each O-D market. The system then uses stochastic processes to simulate actual demand by passenger type for each O-D market. At the same time, the Revenue Management algorithm determines booking limits, in order to optimize revenues.

At the trial level, the sequence followed is explained on Figure 2. Each trial contains 600 repetitions of the same day (sample) with no trends. Bookings are spread over 16 time frames (i.e. booking periods) for each day.

Figure 2: PODS Flow Chart



Source: Zickus, Jeffrey S., *Forecasting for Airline Network Revenue Management; Revenue and Competitive Impacts*, May 1998, p55.

As we described earlier, a historical database is the first thing that is input at the beginning of a trial: at the beginning of a trial, there is no previous flight information to initialize the system and we therefore initialize the database with default values. This is done by using the booking curves initially given as input to the system.

From this point on, the simulation works on a per-sample (i.e. on a per-day) basis, both working on the supply and the demand side at the same time. Within each sample there are 16 booking periods, also referred to as time frames. At the beginning of each time frame, PODS first updates its databases, that is, records the number of bookings that were observed in the previous booking period. With this information now available, it becomes possible for the Revenue Management systems of our simulated airlines to create a forecast of booking requests for the upcoming booking periods and set the booking limits accordingly.

For each passenger, PODS then determines whether the passenger is accepted on a path/fare combination, given availability, and removes these passengers from an

ODF demand accordingly. The availability is then recalculated, taking into account the Revenue Management booking limits and the actual number of bookings. Finally, this new booking information is added to the databases.

Once this process has been completed for each passenger and each time frame, the simulated flight data is summarized and the process is repeated for each of the 600 samples and the 20 trials.

The final output is an average of all the results obtained from the 400 samples that are kept over each of the 20 trials. The output is simply an average of 8000 samples or 8000 repetitions of the booking process for the “same” departure day.

1.1.2 Inputs

As we briefly discussed earlier, PODS requires the user to input a great deal of parameters. Table 1 gives a summary of the major inputs in the Passenger Origin Destination Simulator. These inputs are gathered in the input files according to the structure detailed in the following paragraphs.

Table 1: Major Input Parameters in PODS (See Zickus⁹)

PODS Major Input Parameters		
Airlines (2)	Passenger Types (2)	Seat Optimization Method (RM)
Markets (54)	Fare Classes (4)	Forecasting Method
Booking Curves (2)	Leg Fares (For Leg-based RM)	Detruncation Method
Observations in Forecaster (26)	Base Fare in Market	Probability of Sell-up
Samples (Days - 600)	Restrictions (3)	Capacity on each Leg
Samples Burned (200)	Cancellation Penalty (\$0)	Percentage of Business Passengers
Number of Trials (20)	Cancellation Rate (0%)	Number of Paths per Market
Time Frames (16)	No-show Rate (0%)	Distance on each Leg
Time Frame Reoptimization Days	Denied Boarding Penalty	Number of Virtual Classes and Boundaries (When Applicable)

The first set of data that need to be input in the model is general data, including when the flights become available for bookings (or open), or how many time frames we want to use. In this part of the input file, the number of markets is stipulated. This first set of data also includes the booking curves by passenger type, and whether we model cancellations and no-shows.

Then, we need to determine how many fare types are offered by each airline and with what restrictions. These restrictions are time restrictions - each fare class is available until a given time frame, after which it is closed down and unavailable - minimum stay restrictions - Saturday night minimum stay - or roundtrip or refundability restriction.

The following step involves the type of Revenue Management and forecasting that each of the simulated airlines will be using. In this part of the input file, for each

airline, we must define the algorithm that will be used by the airline, the associated detruncation method and forecasting routine. Other parameters must also be input at this point, including whether the airline decides to account for the possibility of sell-up or not.

The input file then focuses more on the market definition by first detailing all the flight legs that will be used in the network, the associated distances, and the airline that flies each given leg.

The rest of the input file then details each market specifically.

Overall, the input parameters can be divided into three sets of specific types of inputs, as follows.

1.1.2.a System Level Input Parameters

These input parameters we kept constant during our simulations, but can be changed by the user, as any other input in the system. These parameters include the number of airlines, the number of markets, the number of fare classes and so on. For a complete listing of all these parameters, the reader should refer to **Zickus⁹**.

1.1.2.b Airline Input Parameters

These inputs we modified according to the hypotheses we were trying to test. They include the Revenue Management method, the forecasting and detruncation methods and all the related parameters. Again, the reader is referred to **Zickus⁹**.

1.1.2.c Market Level Parameters

Finally, this set of input parameters affects the settings of the markets, the size of demand, the time of day curve, etc. These were not used for our simulations, but the reader can refer to **Zickus⁹** or **Skwarek¹⁰** for more detail regarding these parameters.

The more important parameters to be discussed are the following:

- Number of observations used from the historical databases,
- Number of time frames and their time spacing,
- Fare classes and associated fences, and,
- Demand factor.

⁹ Zickus, Jeffrey S., *Forecasting for Airline Network Revenue Management; Revenue and Competitive Impacts*, May 1998, Chapter 3.1, pp52-64

¹⁰ Skwarek, Daniel K., *Competitive Impacts of Yield Management Systems Components: Forecasting and Sell-up Models*, June 1996

Indeed, the number of observations from the historical databases to be used in the forecaster should be large enough to provide reliable forecasts that are not oversensitive to unusual flights. On the other hand, this number of observations should at the same time not be so large that it incorporates flights that are too old to be of interest. However, this seasonality problem is not taken into account within PODS, and therefore a set of 26 departed flights is a sufficient number.

Second, the number of time frames used in our simulations is 16. The flights open for bookings 63 days before scheduled take-off time. Calculations for the booking limits are then done for each of the 16 time frames. These time frames are weekly time frames until 35 days before departure. They then become bi-weekly, that is, every 3.5 days, until 7 days before departure. At this point, the booking limits are updated every 2 days until 1 day before departure, which is the last update.

Third, the fare classes must be determined for each airline and for each market. In our PODS model, it is assumed that the airline uses a system of four classes – Y, B, M and Q class, where Y class is the highest fare class and is therefore unrestricted. However, the actual fare by class depends on the O-D market, according to a distance formula. The ratios between the fares are 4, 2 and 1.5 between Y and B, B and M, and M and Q, respectively, and remain unchanged whatever the market. The base fare of \$200 for the Q-class 2000 miles market is used to get the fare for the other classes. A factor of 1.6 is applied to this base fare for each doubling of the distance flown on a leg.

The fares in the simulator were chosen to reflect reality as best as possible. In this respect, they do not correspond exactly to fares observed by everyday travelers, but they lead to realistic simulations in terms of seat protections and Revenue Management efficiency. Moreover, the fare ratios are comparable to those observed in the industry.

Table 2: O-D Fares

	O-D Fares	Distance (Miles)							
		500	1000	1500	2000	2500	3000	3500	4000
Fare Class	Y	\$ 320.00	\$ 500.00	\$ 660.00	\$ 800.00	\$ 910.00	\$ 1,020.00	\$ 1,205.00	\$ 1,280.00
	B	\$ 160.00	\$ 250.00	\$ 330.00	\$ 400.00	\$ 455.00	\$ 510.00	\$ 602.50	\$ 640.00
	M	\$ 120.00	\$ 187.50	\$ 247.50	\$ 300.00	\$ 341.25	\$ 382.50	\$ 451.88	\$ 480.00
	Q	\$ 80.00	\$ 125.00	\$ 165.00	\$ 200.00	\$ 227.50	\$ 255.00	\$ 301.25	\$ 320.00

Each fare type then has fences in order to prevent business passengers from having access to these fares. The fences are set up as summarized in Table 3.

Table 3: Fences

Fare Code	Dollar Price	Advance Purchase	Round Trip?	Sat. Night Min. Stay	Percent Non-Refundable
Y	\$100	--	--	--	--
B	\$80	3 day	Yes	--	50 %
M	\$50	7 day	Yes	Yes	100 %
Q	\$40	14 day	Yes	Yes	100 %

The last very important factor to be stressed here is the demand factor. The demand factor is the ratio of the average demand to the aircraft capacity (which was set to 100 in our studies). Hence, a demand factor (DF) of 1.0 means that the demand, on average, will be 100 passengers per flight. On the other hand, a demand factor of 1.2 means that the demand will be of 120 passengers per flight. However, average demand ranges from 60 to 140 depending on the flight, for a system-wide average of 100, at demand factor 1.0. It ranges from 48 to 112– at demand factor 0.8 and from 72 to 168 at demand factor 1.2. In our analysis, we use demand factors 0.8, 1.0 and 1.2 to see how the change in demand influences the results of each Revenue Management method that we will be using. The influence of demand on the results will greatly depend on the type of algorithm used, as the user also decides on the percentage of local vs. connecting passengers in a market, which influences the performance of each Revenue Management alternative. These demand factors were chosen to reflect average system load factors of roughly 70%, 78% and 83% respectively.

1.1.3 Outputs

The output files generated by PODS are comprehensive results of the simulation. They are divided into four sections:

- The first section summarizes the inputs, but does not include the market data,
- The second section holds the intermediate results, that is the revenues per trial,
- The third section is the primary output section, where the revenues and loads per market and leg are recorded, and, finally,
- The fourth section contains supplemental outputs, such as the time frame at which a given flight closed down, etc.

In order to get a clear analysis out of the output files we concentrate on a set of 3 primary measures that will enable us to compare the results of each competitor.

First, the most obvious measure is the overall network revenue of both airlines. This again is the average of the revenues over the twenty trials. While this only gives a first order of magnitude on the performance of the airline's Revenue Management system, it is clearly the more important indicator. However, it does not show differences on a flight basis.

The second indicator then is the load, be it the average network load factor (ALF), the leg load factor, the fare class load mix or even the virtual buckets load mix. These loads give us an indication of the ability of the airline to fill up its planes, and therefore its ability to forecast demand correctly by finding out the loads by fare class. This also provides an explanation of why revenues have varied, depending on the loads of high fare passengers versus low fare passengers.

The third indicator is the fare class closure: it gives the time frame when a given fare class actually closed down. This is a good indicator of the availability of a fare class, on average. The availability depends on the fare class and the demand for a given flight. Ideally, the top fare class should close down upon departure or just before departure, while B class should also remain available longer than M and Q classes, and so on. This would then suggest that the booking limits were very well estimated. However, for a high demand flight, the lower classes will surely close down much earlier than the last available time frame associated with this class. Overall, this indicator lets us gauge whether a lot of passengers were spilled in a given class or not, and whether this lost revenue was offset by the loads in higher classes.

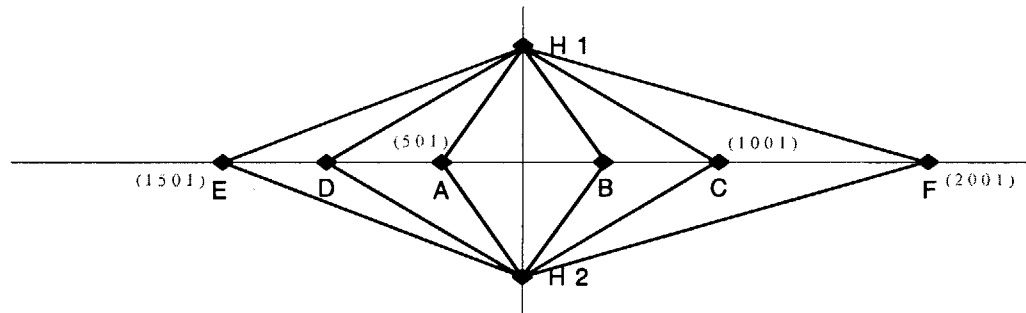
The output files allow us to obtain raw data on the results of a case, data that can then be used to compare the effects of various changes in the airline's Revenue Management system. The three major indicators that we described earlier must be combined to get the best understanding of what really happened, and are not the only tools that we can use. As we briefly discussed, the output files provide us with extensive data on the results of a case.

1.1.4 Network 6B

In order to use PODS, we need to model a network of operations for our competing airlines. In the interest of saving computational power and time because of the intensive passenger choice simulation of 8000 samples, we restricted ourselves to a small network of 6 spoke cities and two hubs, one for each airline, with no inter-hub flights. Moreover, this network has been extensively tested by Lee¹¹ and has proven to be very stable in the results it provides. It is therefore a very good basis for analyzing the impacts of sell-up models on airline revenues. This network also models (in a very simplified way) the current situation within the domestic United States, where a variety of routes and O-D markets are available through a hub. In this case, each airline has its own hub which allows connections to all the spoke cities. This network design was generated and tested by Lee¹¹.

¹¹ Lee, Alex Y., *Investigation of the Impacts of Origin-Destination Control Using PODS*, Network 6B is referred to as Network 3 in the thesis, p55

Figure 3: Network 6B



This set of 6 spoke cities associated with two hub airports includes 54 different origin-destination markets, connected by 24 different flight legs, 12 for each airline. The spatial layout of the network can be seen in Figure 3. There are two spoke cities 500 miles away from either hub, two other spoke cities 1000 miles away from either hub, one spoke 1500 miles away from the hub and one spoke city 2000 miles away from the hub. This enables a range of ODFs from 500 to 4000 miles, while flights range from 500 to 2000 miles.

As for demand, it is segmented into two types – business and leisure.

On a leg basis, demands were set in a systematic way. It was assumed that the short-haul legs and paths have the highest demands, while the longest-haul legs have the lowest demands. The logic behind this attribution of demand is that the higher the demand on short-haul, the more important it becomes to perform Origin and Destination (O&D) Revenue Management. Indeed, not only will the short-haul legs have the lowest fares, as we use a distance based pricing scheme, but this is where the demand bottlenecks will also tend to occur. Therefore, it becomes crucial to have a good Revenue Management system in order to first of all try and keep as many high fare passengers, but also to maximize the gains from displacing a connecting passenger in favor of two local passengers, apparently paying less but actually contributing more to the overall revenues of the airline on a network basis. The issue should become apparent when comparing O-D based Revenue Management systems to leg-based Revenue Management optimizers.

In summary, PODS is a highly detailed simulator that essentially simulates bookings for identical repetitions of a single day, on a competitive network. We briefly discussed earlier the numerous input parameters and the possibilities offered by the simulator. In this thesis, we only use a limited portion of the capabilities of the simulator, on the small network 6B, for consistency purposes.

1.2 Revenue Management Methods

Six different Revenue Management algorithms, or seat optimizers, will be used in the PODS simulations. The goal of this section is to give a brief overview of the

operation of these six methods. More detailed information on each of these six algorithms can be found in **Williamson**¹² and **Lee**⁸. The first five methods we will describe, namely EMSRb, GVN, HBP, DAVN and Netbid, have been extensively described by **Lee**⁸. The last one, ProBP was first introduced by **Williamson**¹² in her prorated EMSR approach and implemented in more detail by **Bratu**¹³.

In the thesis, we will always consider that the fare structure consists in four classes, Y, B, M and Q, from the highest class (Y) down to the lowest (Q).

1.2.1 Fare Class Revenue Management Algorithms

Fare Class Revenue Management algorithms lead to the setting of booking limits on a flight leg, based on forecasts of demand on a leg basis. Therefore, regardless of whether the passenger is traveling via one or more connections, the algorithm nevertheless protects the inventory, i.e. the available capacity, on a flight leg basis. Hence, it is possible that two short-haul passengers will be denied access to a flight in a given fare class in order to enable a long haul, seemingly higher paying passenger, to book the flight. This leads to a few optimization problems and lower revenues than expected. Such Fare Class Revenue Management algorithms detruncate, forecast and protect the inventory on a leg basis. We will explain detruncation and forecasting later on in Part I, Chapter 2:.

1.2.1.a Expected Marginal Seat Revenue (EMSR)

This Expected Marginal Seat Revue algorithm was developed by **Belobaba**¹⁴ (1987). This was one of the first published algorithms for nested booking classes on a flight leg in the passenger airline industry. The algorithm uses leg-based forecasts, by fare class or bucket, and protections.

This rather straightforward Revenue Management model sets the booking limits for each fare class and flight leg, based on a forecast that is also made on a fare class and flight leg basis. The underlying idea is that the uncertainty that a passenger will book a given flight should be taken into consideration when setting the booking limits. Ideally, if demand were deterministic, one could set the protections for each class to its expected demand. However, as this is very far from the truth, this algorithm sets the protections according to the expected marginal revenue of each incremental seat sold in a given class. Therefore, as the probability of selling a seat decreases as the number of seats already booked increases, the expected marginal revenue of the n^{th} seat in the highest class is only a fraction of the original full fare in this class (Y). Hence, the algorithm sets the protection for one class when the next class's full fare equals the expected marginal seat revenue of the n^{th} seat sold in this higher class.

This method follows three basic steps:

¹² Williamson, Elisabeth L., *Comparison of Optimization Techniques for Origin-Destination Seat Inventory Control*, May 1998

¹³ Bratu, Stéphane J-C., *Network Value Concept in Airline Revenue Management*, June 1998, Chapter 3.3

¹⁴ Belobaba, Peter P., *Air Travel Demand and Airline Seat Inventory Management*, May 1987

-
1. Forecast demand on a leg basis, by fare class, including uncertainty, hence with a standard deviation
 2. Determine the protections for each fare class according to the expected marginal revenue of each seat
 3. Get the nested protections for each class, in order to always have the full inventory available to the highest class

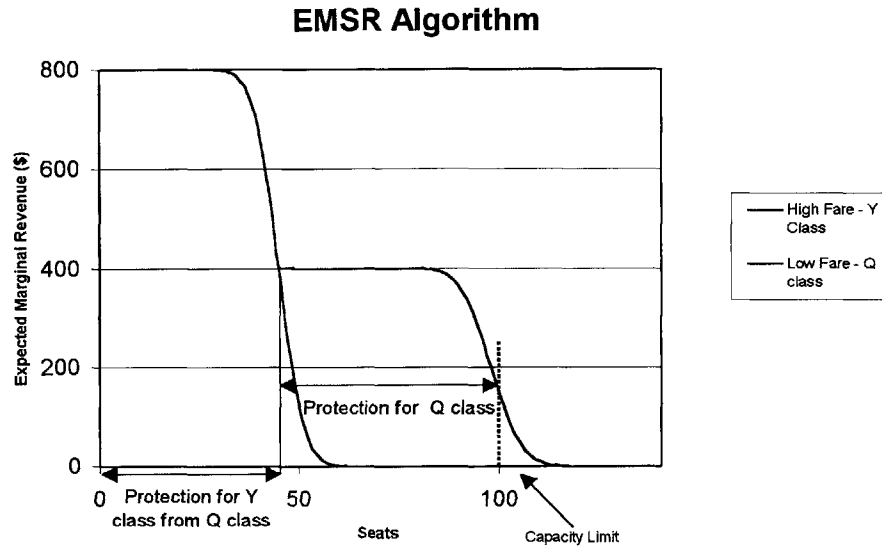
To be a little more precise, another issue that comes up in this method is that protecting one class from the next is sub optimal as, in fact, one should protect one class from all the classes lower. Indeed, it is possible that the EMSR value of the $n+1^{\text{th}}$ seat in the highest class (Y) be still higher than the full fare in M class, while it is already lower than the B fare. Therefore, to deal with this problem in a less computationally intensive manner, the EMSRb¹⁵ version of the algorithm protects the joint upper classes from the class immediately below. For example, if the fare classes are Y, B, M and Q, in descending order, the algorithm will compute the protection for the joint Y and B classes, from M class. The demand estimates are obtained for these joint classes by assuming that demand is normally distributed and independent from class to class. Hence, we simply add the mean demands and add the variances to get the characteristics of the distribution for the lower classes.

Example

In this example, we have a two-class flight. Figure 4 shows the Expected Marginal Revenue of each additional seat sold.

¹⁵ Belobaba, Peter P., *Optimal vs. Heuristic Methods for Nested Seat Allocation*, Presentation to AGIFORS Reservations Control Study Group Meeting, Brussels, Belgium, May 1992

Figure 4: EMSR value of each seat



The idea is to follow the highest curve. First, the probability of selling the initial seats at a high price is close to 1; hence, the expected revenue out of these first seats is close to the full fare. However, as bookings increase, this probability decreases and then, as soon as the highest curve drops below the Low fare curve, we switch to the second curve. As shown on the chart, this gives us the protection for class 1. The transition point between two curves determines the number of seats that should be protected for the associated fare.

Note: This example is valid for both EMSRa¹⁴ and EMSRb¹⁵ models. Determining the protection levels when there are more than two classes becomes more complicated for EMSRa, as we must then determine the protection for one class from all the lower classes. Therefore, in this thesis, we focus on EMSRb and use the algorithm described by Belobaba¹⁵.

Mathematical Explanation

The mathematical logic behind this algorithm is very simple and is as follows:

Determine the value of the protection Π where both curves intersect, according to the following formula:

$$P(\Pi < r) = \frac{f_2}{f_1}$$

Where r is the number of requests for class 1, f_1 and f_2 are the fares for classes 1 and 2 and the requests are normally distributed with a given mean and standard deviation (basic assumption of the model).

In summary, the EMSR algorithms are leg-based Revenue Management algorithms that focus on the expected revenue from the sale of a given seat on the aircraft. These algorithms depend on many assumptions including the distribution of demand (it is assumed to be normally distributed for EMSRb).

1.2.2 Origin-Destination Algorithms

There are various levels of Origin-Destination (O-D) Revenue Management algorithms. By O-D control algorithms we mean any algorithm that allows for different availability for different Origin-Destination pairs. A Revenue Management algorithm is said to be an O-D control method as soon as it involves any type of O-D detruncation, forecasting or protection -- this applies to virtual bucket mapping, which is done on an O-D basis (c.f. Part I, Chapter 1.2.2.a). Hence, among the algorithms used in PODS and in the airline industry we will be seeing all the possible combinations: leg forecast and protection with O-D virtual mapping, O-D forecast with leg protection, leg forecast with O-D protection, and O-D forecast with O-D protection. The last combination we naturally expect to be the most beneficial combination. As we will see in the following paragraphs, there are essentially three types of O-D Revenue Management algorithms. DAVN solves a linear programming (LP) problem in order to ultimately come up with protections for each fare class while GVN uses the EMSR logic. On the other hand, Netbid and Heuristic Bid Price use the concept of a bid price, i.e. set a minimum price that should be paid by the customer in order for him to be allowed to book a seat. The major difference relies in the fact that there is no actual protection in the case of the bid price algorithms and that any passenger paying more than this threshold price can book a seat.

1.2.2.a Greedy Virtual Nesting (GVN)

Greedy Virtual Nesting, a.k.a. VEMSRb (Virtual EMSRb), is an “improvement” on the EMSRb algorithm in that it tries to take into account the fact that it can be more profitable to take a long-haul passenger, thus higher paying passenger, than a short-haul passenger. Therefore, GVN still is leg-based, but uses a set of Virtual Buckets to allocate the fare on a given flight leg. Hence, the mapping of the fares is done on an O-D basis as it ranks the fares from highest to lowest in virtual buckets. Then, the connecting fares, often higher, are ranked higher in the bucketing system and hence benefit from a higher protection on each leg. The protection is, in this case, not made on a fare code basis (a product type) anymore, but rather on a price basis only.

This method obviously allows more low-yield passengers with restricted fares to make the flight. The logic of this algorithm is a “greedy” approach to the problem: Accept the higher paying passengers first, on the basis that they bring more net revenue to the airline. However, as far as yield is concerned, these passengers are lower yield passengers than local passengers, who may have been displaced. In this respect, this method is clearly not revenue maximizing, as it does not account for passenger displacements.

As we will see, it does, however, perform better than EMSRb in general (in Network 6B).

Overall, this Revenue Management algorithm is similar to EMSRb and computes protections in the same way. The major difference is that the fare classes are mapped differently, which results in very different protections. Table 4 gives an example of possible virtual bucket mapping.

Table 4: An Example of Virtual Bucketing

	Published Fare Class	Restrictions	Published Fare	Mapped Booking Class Virtual Bucket	Forecast Detruncation
Local	Y	Unrestricted	\$400.00	Y1	Leg-based
	B	7-day adv.	\$200.00	Y2	
	M	14-day adv.	\$100.00	Y3	
Connecting	Y	Unrestricted	\$1,000.00	Y4	
	B	7-day adv.	\$500.00	Y5	
	M	14-day adv.	\$250.00	Y6	

In this example, we see that the unrestricted Y connecting fare is mapped to the highest Y1 virtual bucket, while the other full Y fare, but local fare, is only mapped into bucket Y3, after the B connecting fare.

In summary, GVN is a leg-based algorithm that is considered O-D based because it actually maps the fares regardless of the O-D and therefore allows for different availability for different O-Ds. However, the forecasting, detruncation, and availability control are all leg-based. GVN has an obvious advantage over EMSRb when flights are not very full in that it will take more connecting passengers. However, when flights are full, this advantage can turn into a big disadvantage as two local passengers bring more revenue than one connecting passenger. Another advantage of this method is that it is transparent to customers, travel agents and competitors.

1.2.2.b Displacement Adjusted Virtual Nesting (DAVN)

Displacement Adjusted Virtual Nesting (DAVN) is one of the few Revenue Management algorithms in PODS to use O-D forecasting. However, it still uses leg-based inventory control. The mechanism behind DAVN involves solving the revenue maximization linear programming (LP) problem described by **Williamson**⁵. The LP problem solved is actually a deterministic LP that allows us to determine the shadow price for each leg in the network. The shadow price is the price that the airline is willing to accept for an additional seat on board a given flight leg. Another way to put it is to say that the shadow price is the expected revenue increase observed from relaxing the capacity constraint. The following step is to set a pseudo fare for each

passenger, in order to account for the displacement cost incurred from booking a seat. For local passengers, the pseudo fare is the actual fare the passenger is paying. For connecting passengers, however, the pseudo fare is the difference between the fare on the leg the passenger is booking on and the shadow price on the other leg (where a local passenger is being displaced). Therefore, for two connecting flight legs i and j , we get the following:

Leg i:	Local Passenger	$PF_L^i = \text{Fare on leg } i$
	Connecting Passenger	$PF_C^i = \text{Fare on leg } i - \text{Shadow price on leg } j$
Leg j:	Local Passenger	$PF_L^j = \text{Fare on leg } j$
	Connecting Passenger	$PF_C^j = \text{Fare on leg } j - \text{Shadow price on leg } i$

Where PF stands for Pseudo-Fare.

This example was in the case of a two-leg connection. For multi leg paths, the same method is used with more shadow prices, the sum of the shadow prices on the other legs being subtracted from the fare on the leg studied for a connecting passenger.

At this point, the optimization algorithm used is GVN after mapping all the new Pseudo-fares into the system. Using the same example as earlier, we get the following mapping:

Table 5: Mapping the Fares with DAVN

	Published Fare Class	Restrictions	Published Fare	Pseudo Fare	Mapped Booking Class Virtual Bucket	Forecast Detruncation	Protection
Local	Y	Unrestricted	\$400.00		Y1	O-D based	Leg-based
	B	7-day adv.	\$200.00		Y2		
	M	14-day adv.	\$100.00		Y3		
Connecting	Y	Unrestricted	\$1,000.00	\$800.00	Y4		
	B	7-day adv.	\$500.00	\$300.00	Y5		
	M	14-day adv.	\$250.00	\$50.00	Y6		

In summary, DAVN performs O-D forecast and data collection, while it still remains a leg-based algorithm in terms of protection. As was established by Lee⁸, this algorithm proves to be amongst the higher performance Revenue Management algorithms tested in the PODS simulations.

1.2.2.c Deterministic LP Network Bid Price (Netbid)

Netbid uses the notion of bid prices to determine whether a passenger should be allowed to book a flight or not. Unlike the EMSR methods and the related methods (EMSRb, GVN and DAVN), the bid price methods base the booking of a passenger

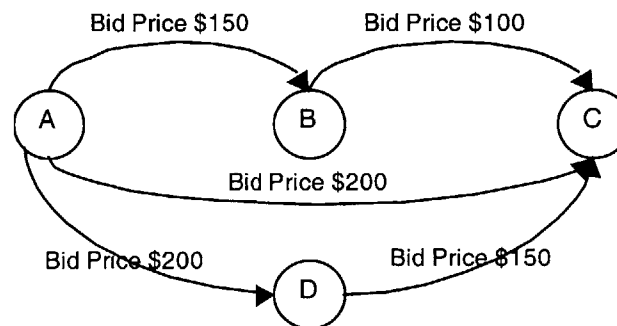
on the price he (or she) pays, compared to a minimum price. The algorithm determines dynamically a minimum price that should be paid by a passenger in order to be able to board the flight.

In the case of Netbid, the same deterministic LP is solved as for DAVN, which again involves O-D forecasting and detruncation, based on ODF historical data. This LP yields the shadow prices for each leg in the network. These shadow prices are then directly used and compared to the price the passenger is willing to pay in order to determine whether he should be allowed to book the flight. This optimization then is done on an O-D basis. Indeed, for local passengers, the fare offered to the passenger is compared to the bid price on the leg. For connecting passengers, the fare paid by the passenger should be greater than the sum of all the shadow prices on the legs traversed.

In summary, this algorithm can be considered to be a fully O-D based algorithm, as it both forecasts on an O-D basis and optimizes seat allocation on an O-D basis.

The following example shows a very simple network where the bid prices for each leg have been computed. Then, if a passenger requests a seat on leg A-B, he will be accepted only if he is willing-to-pay more than \$150. Another point to stress here is that different paths may have very different bid prices. For example, to get from A to C, passengers can fly nonstop, or connect through B or D. The nonstop flight has a bid price of \$200, while the A-B-C connection has a bid price of \$250 and the A-D-C connection has a bid price of \$350. These differences reflect the convenience of each path, as they are more or less demanded.

Figure 5: Netbid Bid Prices



1.2.2.d Heuristic Bid Price (HBP)

This HBP method was developed by **Belobaba**¹⁶. The idea, again, is to account for the displacement of local passengers by connecting passengers. This bid price method uses the same logic as Netbid. However, there are a few crucial differences. Forecasting, detruncation and optimization are all done on a leg-basis. HBP uses the

¹⁶ Belobaba, Peter P., *The Evolution of Airline Yield Management: Fare Class to Origin-Destination Seat Inventory Control*, Handbook of Airline Marketing, 1st Ed., 1998, Chapter 23, pp. 285-302

same data collection and forecasting principles as GVN and only differs from GVN in terms of optimization and availability control.

This bid price is calculated based on the EMSR value of the last seat available on a flight for the local passengers. However, for connecting passengers, the bid price becomes the sum of this value and the product of the percentage of local passengers on both legs and the EMSR value of the last seat available on the connecting flight. For a simple two-leg case, the bid prices are connected as follow:

Leg i: $BP_{Li} = EMSR_{Ci}$

$$BP_{Ci} = EMSR_{Ci} + d \cdot EMSR_{Cj}$$

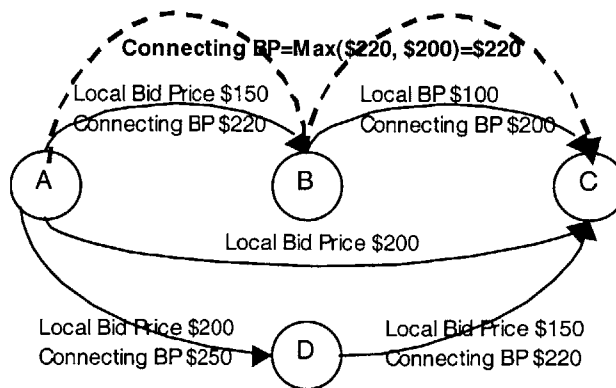
Where i is the index of the leg, $EMSR_c$ is the critical EMSR (EMSR value of the last available seat), d is the product of the estimated percentage of local passengers on each leg, and BP the Bid Price for local (L) or connecting (C) passenger.

Once again, to determine whether a passenger may book a seat on a flight, his (her) fare is compared to the bid price on each leg. For connecting passengers, the fare must be higher than the higher of both bid prices on each leg. In terms of mathematical formulation, we accept a connecting passenger if and only if the fare F that he (she) pays is such that:

$$F \geq \text{Max}(BP_{C,i}, BP_{C,j})$$

For the simple network example that we used earlier, the bid prices would look a little more complicated, as shown on Figure 6. For a passenger flying from A to C on the connecting flight through B, the airline will accept the booking if and only if the fare paid by this passenger is greater than the maximum of the two connecting bid prices, that is, \$220.

Figure 6: HBP Bid Prices



A rather disturbing issue regarding bid prices is that this type of algorithm does not protect seats and offers no control over the number of bookings in between

recalculations of the bid prices. However, if bid prices are recalculated with reasonable frequency, this should not be an issue.

Once again, as GVN, HBP is a fully leg-based algorithm that uses virtual mapping of the fares on a network level. For this reason, we consider HBP as an O-D Revenue Management method.

1.2.2.e Prorated Bid Price (ProBP)

Finally, the last and most recent Revenue Management algorithm tested and used in PODS is ProBP. This method was thoroughly examined and tested by **Bratu**¹³. The underlying idea behind this method is again to use a bid price method to determine whether a passenger should be allowed to book on a given flight. However, the algorithm now determines the bid price for each leg depending on the O-D and splits up the actual total fare in between the legs traversed, for connecting passengers. This concept takes into account the structure of the network and is a zero-sum algorithm that takes into consideration the demand on each leg.

The method used to compute the value of each prorated fare is the following:

$$\sum_{m \in L_j} EMSR_c(m) \neq 0 \Rightarrow PRF(j, k) = \frac{EMSR_c(k)}{\sum_{m \in L_j} EMSR_c(m)} \times F_j$$

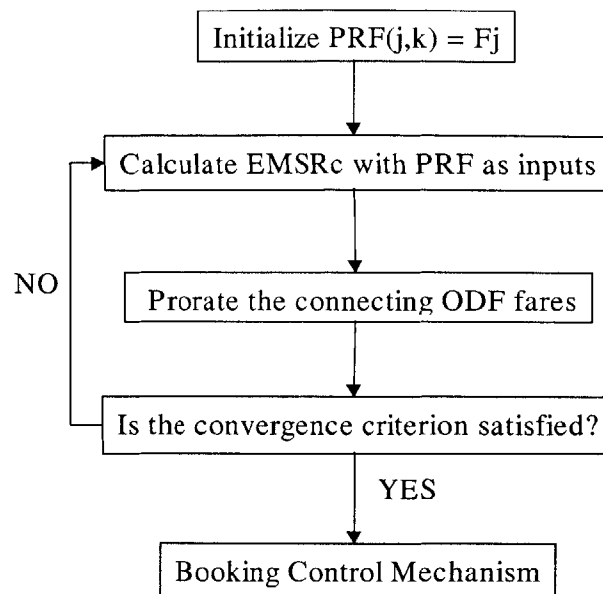
$$\sum_{m \in L_j} EMSR_c(m) = 0 \Rightarrow PRF(j, k) = \frac{F_j}{card(L_j)}$$

Where PRF(j,k) is the prorated fare of ODF j on leg k
 L_j is the set of legs traversed by ODF j
 F_j is the original fare of ODF j
 EMSR_c (m) is the critical EMSR value of leg m (i.e. the EMSR value of the last seat sold on leg m)

As we discussed earlier, this algorithm takes into account the demand on each leg through the use of the EMSR_c values for the proration of the fare.

Another issue that arises in the calculation of these prorated fares is that the EMSR_b model uses the total itinerary fare of the ODF traversing leg k to compute the EMSR_c value, hence overestimating it. A convergence model was therefore developed to address the problem, as shown on Figure 7.

Figure 7: ProBP Logic



This algorithm was shown to bring significant improvement to the revenues of the airline using this new algorithm. The gains obtained through this algorithm were substantially higher than those of any other Revenue Management method previously described.

1.3 Summary

This introduction to PODS and the Revenue Management methods allowed us to understand how the simulator itself works. Moreover, we introduced and explained in detail the notion of Revenue Management and the algorithms used by PODS. EMSRb, GVN and HBP are fully leg-based algorithm, while all the others combine either O-D forecasting or O-D optimization, or both. GVN and HBP are considered O-D Revenue Management methods because they involve mapping all the fares into virtual buckets. The other distinction between all these Revenue Management methods concerns the type of protection (or not) that is used. Indeed, in the case of bid prices, there is no physical protection of the seats, that is, no actual limits set, only a minimum price that must be paid by the customer.

Table 6 summarizes the different types of Revenue Management systems we just introduced and their specifications.

Table 6: Summary of Revenue Management Algorithms Specifications

Revenue Management Algorithm	Data Collection and Forecast	Control Type	
		Leg/O-D	Limit/Bid
EMSRb	Leg-based	Leg-based	Limit
GVN	Leg-based	Leg-based	Limit
DAVN	Path-based	Leg-based	Limit
Netbid	Path-based	Path-based	Bid
HBP	Leg-based	Path-based	Bid
ProBP	Path-based	Path-based	Bid

O-D algorithms

Chapter 2: Detruncation and Forecasting

In this chapter, we discuss detruncation and forecasting methods. Both these methods contribute to generating a forecast of demand. However, detruncation differs from forecasting in that it is applied at an earlier stage of the overall process. Detruncation unconstrains demand, that is, generates an estimate of the actual demand for a departed flight that was closed (i.e. full). As we will explain in more detail, once a flight is full, the only information the airlines get is that the demand was at least equal to the capacity of the aircraft. Forecasting, on the other hand, consists of getting an estimate of future demand, given previous demand, including the previously estimated unconstrained demand.

Both these tools are critical to reliable forecasting, as we will explain in the following paragraphs. We will focus on detruncation methods, as this is one of the aspects studied in this thesis. We will, however, also discuss forecasting algorithms in a second part.

2.1 Detruncation Methods

Detruncation is the process of estimating the unconstrained demand, given that, in the passenger airline industry, we have to deal with fixed aircraft capacities. Therefore, in the event that a flight is full, it becomes very important, for forecasting reasons, to be able to estimate the actual demand. As soon as requests are refused, it becomes impossible to infer the actual demand for a flight directly from the bookings. Therefore, it becomes necessary to find a way to estimate this demand. Indeed, it will then allow the Revenue Management system to make more accurate predictions and to set adequate protections. Another alternative would have been to base our forecasts only on unclosed observations, where we can accurately know demand on each flight. However, this is obviously not satisfactory, as it does not account for high demand periods and constantly underestimates unconstrained demand. Therefore, the basis for unconstraining demand is to use historical booking patterns and then to apply these patterns to our closed observations in order to reevaluate the actual demand for a given flight.

In short, there are two possibilities that can occur:

- The flight is not full in any class, or,
- The flight reaches capacity in at least one fare class.

In the first case, we do not need to use our detruncation algorithms, as demand can be inferred from the number of bookings. In the second case, however, we need to

apply our detruncation algorithms to our closed observation(s) in order to get an estimate of the demand and its variance. The reader is referred to **Skwarek**¹⁰ who tested scenarios of one airline using no detruncation in its Revenue Management system while the other airline did. The results of this analysis show that even at low demand factors, the impacts of the detruncation models on revenues are as high as 3.5%.

In this chapter we present various detruncation methods that we will be using in PODS: Booking Curve detruncation, Projection detruncation and Adjusted Booking Curve detruncation.

2.1.1 Booking Curve Detruncation

In the next paragraphs, we describe how Booking Curve detruncation allows the user to obtain an estimate of the unconstrained demand on a given flight and fare class. We first describe a critical assumption of the model and then get into a more detailed explanation of the algorithm.

2.1.1.a Underlying Assumption

Booking curve detruncation is the first and easiest method used in PODS to estimate unconstrained demand. The idea behind this method is that bookings follow a similar pattern for all flights. Therefore, it is a good approximation to suppose that fourteen days before departure, for example, 40 percent of the people wanting to take a given flight will have already booked that flight. Hence, this detruncation method simply uses the unclosed flights to estimate how much more people (percentage-wise) book from one time period to the last time period and use these measures to evaluate the unconstrained demand on closed flights.

2.1.1.b The Algorithm

As briefly described above, the Booking Curve detruncation algorithm computes the expected increase in relative bookings from one booking period to the next, and, given these values, computes the relative overall variation in bookings from one period to the last period. This gives us a booking curve, which is used to determine the unconstrained demand for closed flights.

The advantage of this method is that it is a time dependent and relatively easy method. Indeed, the detruncated demand will depend on the period when the closed flight actually closed down. For example, if a flight closed down at time period 2, then we may infer from the booking curve, for example, that the bookings represent only 35% of the total demand. If that same flight closed at time period 5, we may then assume that we had almost 70% of the total demand. Therefore, we get a different value for the unconstrained demand as a function of when the flight closed down.

Given the available databases at most airlines, it is possible to generate a booking curve for each fare class and flight. This is done by analyzing the “pick-up” (number of passengers booked from one time period to the next) in each time interval for the unclosed observations. The booking curve detruncation algorithm then calculates the ratio of bookings in each interval to the number of bookings in the preceding time interval for a given fare class on a flight. Given these pick-up ratios, the algorithm then builds a booking curve and uses it to detruncate and therefore estimate the demand for closed observations.

For more detailed explanations of the algorithm, the reader is referred to **Zickus**⁹.

2.1.2 Projection Detruncation

In this section, we follow the same pattern as for Booking Curve detruncation by first describing the critical assumptions of the model, then explaining the model in more details and by finally providing an example.

2.1.2.a Underlying Assumptions

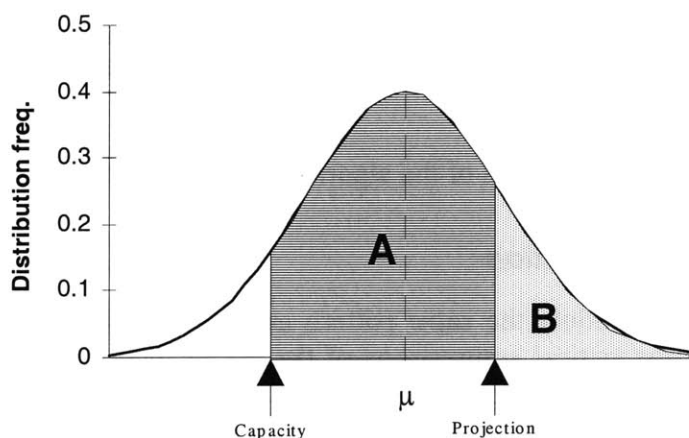
This algorithm was developed by **Hopperstad** and is based on the assumption that the conditional probability that we underestimate the unconstrained demand for a flight and fare class, given that this fare class closed down, is a constant value. We get into a little more detail in the following paragraphs, but the reader should refer to **Zickus**⁹ for extensive explanations of the algorithm.

2.1.2.b The Algorithm

The algorithm relies on the assumption that demand is normally distributed. Given this assumption, the method first computes the average demand over the unclosed observations. However, this average demand is clearly wrong, as it does not take into consideration the closed observations.

At this point, the method uses an arbitrary value, τ , to evaluate the detruncated value of the closed observation. The closed observation is projected to a new value such that the ratio of the area to the right of this new value to the area to the right of the original value is equal to τ . This is clearly explained on the following graph.

Figure 8: Projection Detruncation



$$\tau = \frac{B}{A + B}$$

From this point on, we then use all the observations – the old unclosed observations and the newly projected observations – to compute the mean and the standard deviation of demand. We repeat this same process for the originally closed observations until the projections (and the mean demand and standard deviation) converge. At this point, we have detruncated and obtained unconstrained values for demand.

This process is repeated at each of the 16 time frames defined in PODS. Therefore, we effectively detruncate at each time frame, which leads to different estimates of the unconstrained demand depending on when a given fare class closed down.

This method can be better understood in the following example from the **AGIFORS** meeting in Switzerland in 1987.

2.1.2.c An Example: AGIFORS 1987

Table 7: Application of Projection Detruncation

Tau = 0.3				
Obs	Bookings	Step 1	Projection 1	Projection 2
1	26	26	26	26
2	7		23.2	24.9
3	22		27.3	27.6
4	20	20	20	20
5	14	14	14	14
Mean		20	22.1	22.5
Std dev		6	5.3	5.5

In this example, we have five observations, two of which closed down before take-off, observations 2 and 3. The τ value was set to 0.3. The first step is then, as we discussed earlier, to compute the mean and standard deviation of the unclosed observations. Here, the mean is 20 and the standard deviation 6. We then use the τ value to project the two closed observations, as shown in “projection 1” in Table 7 above. Again, we compute the mean and standard deviation of this new set of observations, and apply the projection again, to the **original closed observation values**. We repeat this final step until the mean and standard deviation of the distribution converge. The mean and standard deviations thus obtained converge after a few iterations.

2.1.2.d Previous Results

In his thesis, Zickus⁹ extensively tested the effect of forecasting and detruncation algorithms on airline revenues in the same network 6B environment. His findings are that the “best” τ value is 0.15, when airline A uses a combination of Pick-up forecasting (c.f. 2.2.1 below) and Projection detruncation while airline B uses Pick-up forecasting with Booking Curve detruncation. This is true for EMSRb and GVN only. The other Revenue Management algorithms behave differently. However, in this thesis, we focus on EMSRb and GVN algorithms, and will therefore refer to this $\tau=0.15$ case as the “base case” for Projection detruncation.

Zickus⁹ goes much deeper into the details of why such results are observed. The reader is therefore referred to Zickus’s thesis for more detail regarding the influence of Projection detruncation on both the EMSRb and the GVN Revenue Management algorithm.

2.1.3 Adjusted Booking Curve

In this last section regarding detruncation methods, we focus on Adjusted Booking Curve detruncation. In the first paragraph, we explain the difference between this method and “regular” Booking Curve detruncation. In a second paragraph, we briefly present the results of previous research work concerning the impact of this detruncation method as a function of the adjustment.

2.1.3.a Variation of the Booking Curve Detruncation Method

This method is directly inspired from the booking curve detruncation algorithm. Booking curve detruncation tends to lead to a biased estimation of the booking curve. Indeed, low first period bookings will lead the forecaster to protect fewer seats for the concerned class. Therefore, if bookings then get back to normal, or are unexpectedly high in the second period (assuming there are only two periods), the fare class will close down, leading to a closed observation. Hence, this observation will not be used in the building of the booking curve. Overall, the booking curve will then be biased high, on average. This is better shown on the following example:

Table 8: Bookings are biased high

	Yclass bookings	
	Period 1	Period 2
flight#1	50	50
flight#2	40	60
flight#3	60	40
	55%	100%

Most Likely to be Closed

Hence, the underlying idea in Adjusted Booking Curve Detruncation is to adjust the booking curve (obtained through the booking curve detruncation algorithm) slightly to correct for this high bias effect.

2.1.3.b The correction

The model used to account for this high bias and to correct the booking curve is a simple linear transformation applied to the initial booking curve results. This model uses the same methodology as the Booking Curve Detruncation algorithm, and then applies a correcting factor to the results, as follows:

Pbscale: Scaling Parameter ($\in [0, 1]$)

Cumb (t): Cumulative Percent Bookings at Period t (estimated from open observations)

NCumb (t): New cumulative percent bookings at period t

- If $\text{Cumb}(t) < 50\%$ then:

$$\text{NCumb}(t) = \text{Pbscale} \times \text{Cumb}(t)$$

- If $\text{Cumb}(t) \geq 50\%$ then:

$$\text{NCumb}(t) = 1 - (1 - \text{Cumb}(t)) \times (2 - \text{Pbscale})$$

This simple linear correction allows us to get lower estimates for the entire curve, with maximum effect at 50% bookings. The following charts show examples of the correction effect booking curves. The first chart has a scaling factor of 0.9, the second chart, a scaling factor of 0.6.

Figure 9: Pbscale with Scaling Factor 0.9

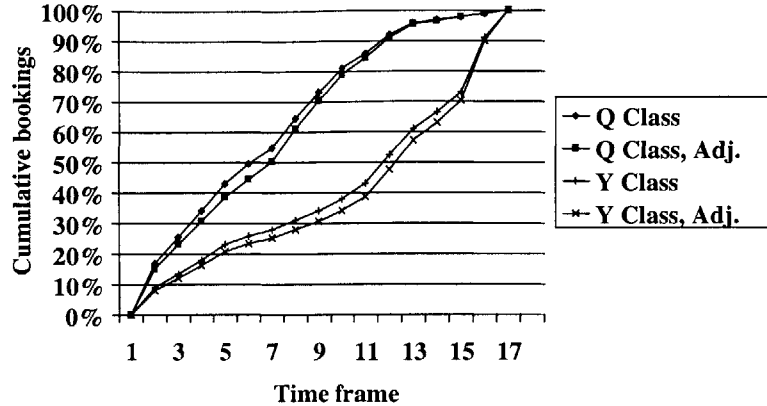
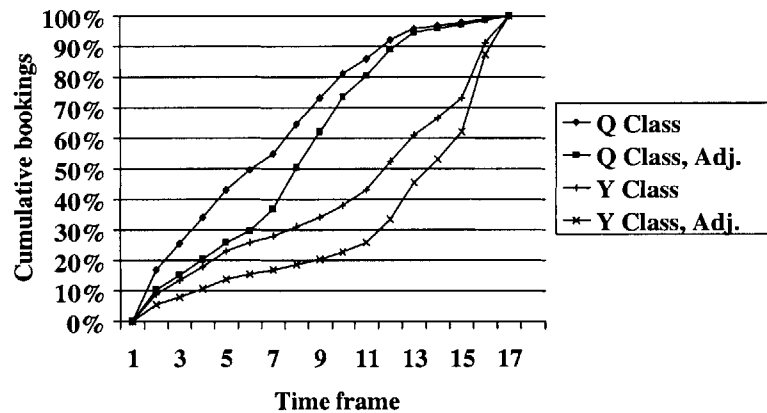


Figure 10: Pbscale with Scaling Factor 0.6



Obviously, the effect of the scaling factor increases as the scaling parameter decreases and approaches 0.

2.1.3.c Best Scaling Factor

Tests have been run with this detruncation method and have shown that the best results are obtained with very different Pbscale factors, depending on the competitive environment and the level of demand. When only one airline changes detruncation methods, the best scaling factor is somewhere between 0.4 and 0.5 (c.f. **Bratu**¹⁷), again depending on the demand factor. Hence, it appears here that the Booking Curve Detruncation model can be greatly improved in some cases by the use of the scaling factor. However, it must also be pointed out that the scaling factor does not always improve the performance of the airline.

¹⁷ Bratu, Stéphane J-C., *Modified Booking Curve Detruncation and Sell-up Techniques*, PODS Summit IX Presentation

2.1.4 Summary

There are a number of detruncation methods that are currently used by Revenue Management systems in the airline industry. We briefly described the more common ones in the previous paragraphs. PODS is setup to use any of the previously described methods. Our base case scenario uses the Booking Curve Detruncation method to estimate unconstrained demand.

2.2 Forecasting: Two Methods

Forecasting, as opposed to detruncation, is the process of finding a quantitative estimate of the likelihood of future events, based on past and current information. In short, in the passenger airline industry, we are trying to get an estimate of the future demand for a flight, based on similar past flights and current bookings on the flight of interest. Forecasting can be done on various levels ranging from a macro level (forecast for the entire network for a year) to a micro level (forecast on a given leg and fare class). Revenue Management requires mainly micro forecasting. In particular, we are concerned with leg and fare class forecasts or ODF forecasts, depending on the type of Revenue Management system we use.

The forecasting algorithms are a necessary part of Revenue Management algorithms. Indeed, it is necessary for the system to have a forecast of future demand to be able to set booking limits. The forecasting method is also re-optimized at every time period, in order to constantly adjust the forecast to the latest available information.

The PODS software offers two possible forecasting methods:

- Pick-up forecasting, which is the more commonly used method in this thesis
- Regression forecasting

We will describe these two methods briefly in the following paragraphs.

2.2.1 Pick-up forecasting

Pick-up forecasting is similar to booking curve detruncation in the method used to forecast. Moreover, the pick-up forecasting method has similarities to a regular time series, yet is more detailed. Indeed, instead of only using an average of the previously observed bookings or unconstrained demand for previous flights, this method also includes the number of passengers picked up from one time period to the next. For the purpose of our Revenue Management methods, the forecasting is done on a fare class and path or leg basis, depending on the Revenue Management method used.

The forecasting is done by averaging the pick-up from one time period to the next for a fixed number of previous flights. This average is then added to the current (or

forecast) number of bookings for the current time period. More details on the algorithm can be found in **Zickus's** thesis⁹.

The following example allows better understanding of the algorithm.

Table 9: Historical Bookings on Hand

DAYS OUT	0	7	14	21	28	35	42	49	56
MAR 05	72	42	24	20	16	10	13	10	4
MAR 12	58	22	19	18	12	11	7	7	5
MAR 19	61	30	19	15	9	5	4	2	2
MAR 26	43	28	19	13	8	4	0	0	0
APR 02	64	36	23	13	5	4	4	3	1
APR 09	62	39	28	18	17	14	10	9	8
APR 16	46	17	8	6	6	2	3	3	3
APR 23	49	26	15	10	6	5	4	3	0
APR 30	-----	20	8	6	3	2	0	0	0
MAY 07	-----	-----	17	16	8	3	3	3	2
MAY 14	-----	-----	-----	7	4	1	0	0	0
MAY 21	-----	-----	-----	-----	4	3	2	0	0
MAY 28	-----	-----	-----	-----	-----	2	2	2	2
JUN 04	-----	-----	-----	-----	-----	-----	4	2	2
JUN 11	-----	-----	-----	-----	-----	-----	-----	0	0
JUN 18	-----	-----	-----	-----	-----	-----	-----	-----	3

Table 9 shows the bookings for a set of eight flights departed between April 23rd and March 5th. We assume in this case that the flights were not closed down so we do not have to deal with detruncation problems. Given all these bookings from one time period to the next, we now use this information to forecast the final bookings for the May 14th flight, for example.

First of all, we compute the pick-up from one period to the next by simply subtracting the bookings in one time period from the previous time period's bookings. We then get the following pick-ups:

Table 10: Pick-up

Pick Up	7 to 0	14 to 7	21 to 14	28 to 21	35 to 28	42 to 35	49 to 42	56 to 49
MAR 05	30	18	4	4	6	-3	3	6
MAR 12	36	3	1	6	1	4	0	2
MAR 19	31	11	4	6	4	1	2	0
MAR 26	15	9	6	5	4	4	0	0
APR 02	28	13	10	8	1	0	1	2
APR 09	23	11	10	1	3	4	1	1
APR 16	29	9	2	0	4	-1	0	0
APR 23	23	11	5	4	1	1	1	3
APR 30	-----	12	2	3	1	2	0	0
MAY 07	-----	-----	1	8	5	0	0	1
MAY 14	-----	-----	-----	3	3	1	0	0
MAY 21	-----	-----	-----	-----	1	1	2	0
MAY 28	-----	-----	-----	-----	-----	0	0	0
JUN 04	-----	-----	-----	-----	-----	-----	2	0
JUN 11	-----	-----	-----	-----	-----	-----	-----	0
JUN 18	-----	-----	-----	-----	-----	-----	-----	-----

The second step in forecasting the pick-up from day 21 to day 0 for the May 14th flight (for which we already have the bookings on hand on 21 days before departure)

we then need to get an average pick-up for each set of pick-ups. For example, here, we will average the pick-ups over eight observations. We then get the following average pick-ups (Table 11):

Table 11: Average Pick-up and Standard Deviation

Pick Up	7 to 0	14 to 7	21 to 14
MAR 05	30	18	4
MAR 12	36	3	1
MAR 19	31	11	4
MAR 26	15	9	6
APR 02	28	13	10
APR 09	23	11	10
APR 16	29	9	2
APR 23	23	11	5
APR 30		12	2
MAY 07			1
MAY 14			
MAY 21			
MAY 28			
JUN 04			
JUN 11			
JUN 18			
Average Pick-Up	7 to 0 26.875	4 to 7 10.625	21 to 14 5.25

Given these average pick-ups, we can simply add them up and then add them to the current number of bookings to get the forecast for the May 14th flight, 21 days before departure. It is also possible to include a measure of the standard deviation, to get an idea of the spread of the distribution.

Therefore, overall, our forecast for the May 14th flight is 49.75, as we already have 7 bookings on hand 21 days before departure.

L'Heureux¹⁸ at the 26th AGIFORS meeting in 1986 developed a more general method involving weighting of observations. The advantages of this modified pick-up model include the possibility of placing more weight on the most recent flights and taking into account the most current available data. Therefore, the impact of odd flights on the forecast is reduced. However, this method is more subject to odd periods of booking activity. Overall, the basic method and the more sophisticated ones lead to similar amounts of error.

We stress here that PODS only uses the simplest version of this general formulation of the Pick-up algorithm. PODS only averages pick-ups over departed flights to evaluate the pick-up for a certain period of time. Unlike this more complicated

¹⁸ L'Heureux, Ed, *A New Twist in Forecasting Short-term Passenger Pickup*, 26th AGIFORS Symposium, October 1996

formulation, PODS does not take current flights into consideration and does not weigh the flights in any way.

2.2.1.a Important Note

There are a number of assumptions that were made in the example that should be discussed briefly here. First of all, we used only data on flights from the same day of week. Indeed, it is crucial to keep in mind that different day of the week demands behave differently and that this should be taken into consideration when forecasting demand. Moreover, there also is the problem of seasonality that, even if it is not very important for leisure passengers, has a heavy influence on business traffic. Hence, we must be very careful when it comes to building our historical database for the forecast. Too many flights will lead to seasonality problems, and too few will have too low a correlation.

Because PODS was designed as a stationary process, it does not account for seasonality issues or the like. Hence, there are no such issues to be taken into consideration when using forecasting in the software. PODS uses data from 26 previous flights. It was chosen to use 26 data points in order to combine good reliability with realism. Indeed, too few data points would not allow for a robust forecast while too many would make it impossible to realistically encompass seasonality issues and the like, were we to apply this method in the real world.

2.2.2 Regression Forecasting

Regression forecasting uses the straightforward technique of least-square regression. In this case, after detruncation methods have been applied to the closed observations, the forecaster then comes into play and uses the observations to forecast demand on future flights.

Unlike pick-up forecasting, however, this method uses only flights that have departed to forecast future flights.

The method tries to relate the number of bookings accumulated at a given point in time for a given flight to the final bookings. The basic mathematical formula used in this method is the following:

$$X_{F,i} = \alpha_n + \beta_n X_{F,i-n} + \varepsilon_n$$

Where

- $X_{F,i}$ is the total number of bookings after time frame i ,
- n is the number of time frames over which the model is calculated,
- α_n and β_n are the intercept and slope of the linear regression model for time period i , and,

-
- ε_n is the error in the model.

We will not get into the details of this method since we are not using it in our analyses, but the reader can refer to **Zickus**⁹ for more detail.

2.3 Objective of Detruncation and Forecasting

To summarize this chapter discussion of detruncation and forecasting, we will concentrate on the benefits obtained from detruncation and forecasting in the field of Revenue Management.

Besides the obvious use of being able to forecast demand for a given flight on a certain day of the week, at a certain time of day and during a specific season, there are a number of benefits that can be obtained, in the airline industry, from these detruncation and forecasting techniques. Moreover, it is clear that better forecasting will allow the airlines to better match their available capacity to the observed demand, and therefore increase their profits, as we will discuss in the following chapters. In particular, it has been established by **Lee**¹⁹ in 1990 that a 10 percent increase in the forecast accuracy can bring a 10 to 60 million-dollar increase in annual revenue for a major US airline. Overall, **Lee's**¹⁹ conclusion is that forecasting accuracy is extremely important and all the more important as the demand increases.

¹⁹ Lee, Anthony O., *Airline Reservations Forecasting – Probabilistic and Statistical Models of the Booking Process*, September 1990

Chapter 3: Sell-up

The goal of this thesis is to explain how accounting for sell-up in airline Revenue Management can lead to increased revenues for the airline, and how this influences the competitor's revenues. Before we get into any results, let us first explain what sell-up is.

Let us consider a passenger who wants to travel from city A to city B on a given day and at a given time. This passenger calls the airline, or his travel agent and requests a seat on this particular flight (typically, a passenger will call the airline and give his (her) time window and let the reservation agents give him the corresponding schedule), within a given price range. Assuming that there are no seats available within the passenger's price range, he (she) then has the following possibilities:

- Try another flight in the same market, at his desired price or fare class.
- Try another flight in the same market, in his desired fare class, but on a different airline.
- Cancel his (her) travel plans.
- Decide to pay more money for the same flight and inquire about the next higher fare class.

As the reader recalls from the introduction, this final behavior is what we referred to as sell-up. Depending on this passenger's time and price sensitivity, he (she) may decide to sell-up or not.

3.1 Understanding the Mechanisms Involved

In this paragraph, we will explain why it is reasonable to expect some passengers to be willing to sell-up and why sell-up occurs. In particular we will examine passenger behavior as it is generally modeled in the airline industry. We will then very briefly talk about pricing schemes that relate to this notion of sell-up and discuss how one could monitor the occurrence of sell-up in the real world.

In her 1990 Master's thesis, **Bohutinsky**²⁰ points out that sell-up can be viewed not only as a behavior of passengers but also as a means of increasing airline revenues. Indeed, when a passenger is denied his or her first choice of fare class, the reservation agent will always try to offer a satisfactory alternative. Among these

²⁰ Bohutinsky, Catherine H., *The Sell-up Potential of Airline Demand*, 1990

alternatives, there often is the possibility for the passenger to pay a higher price and book a seat on the flight he or she was originally targeting. Should the passenger accept this alternative, the airline increases its revenues by the fare difference. In reality, what happened is that the passenger was spilled from his or her original choice to be immediately recaptured by the same airline in another fare class or on a different path from the origin to the destination city. In all cases, this is to the benefit of the airline that either:

- Increased its revenues by selling a more expensive ticket to the passenger in a higher class on the same flight, or,
- Increased its revenues by selling a seat on another flight (if the Revenue Management system works well, the airline should be expecting to sell all other high fare seats on the original flight as it denied access to this passenger).

In this view, it is undoubtedly beneficial to try and push all passengers willing to sell-up to a higher fare class. Therefore, it becomes interesting to evaluate the sell-up rate, that is, the percentage of people who are willing to sell-up. The **Air Transport Committee of the Canadian Transport Commission (CTC)** performed a direct survey of domestic (Canadian) scheduled passengers using deeply discounted fares²¹. The survey asked these passengers what they would have done if their fare had not been available. Even though this survey did not focus on a specific airline, it provided insight on passenger behavior and the probability of sell-up.

As **Bohutinsky**²⁰ states, the best way to estimate sell-up would be to directly monitor reservation calls. In doing so, the airlines could observe passenger behavior and record choices as different alternatives are offered to the passengers. Moreover, since airlines today try to reduce the number of tickets purchased through travel agents, this method could prove useful. However, monitoring all reservation calls would be very costly in terms of time and resources. Moreover, this method could also prove very unreliable, as many passengers tend to make multiple reservations on different carriers in order to compare offers and then cancel the alternatives that do not satisfy them.

Overall, it is clear that there is revenue to be gained from better understanding the sell-up phenomenon but also the risk of losing revenue if the probability of sell-up is not correctly estimated. **Bohutinsky**²⁰ provides more detail regarding the possible revenue gains or revenue impact difference of accounting for sell-up.

3.1.1 Passenger Behavior: Leisure vs. Business

The phenomenon of sell-up is closely related to the differences among passenger types. Indeed, different passengers have different behaviors and these behaviors include the possibility of sell-up in different ways. Therefore, to better understand the sell-up phenomenon, we need to describe the different passenger types.

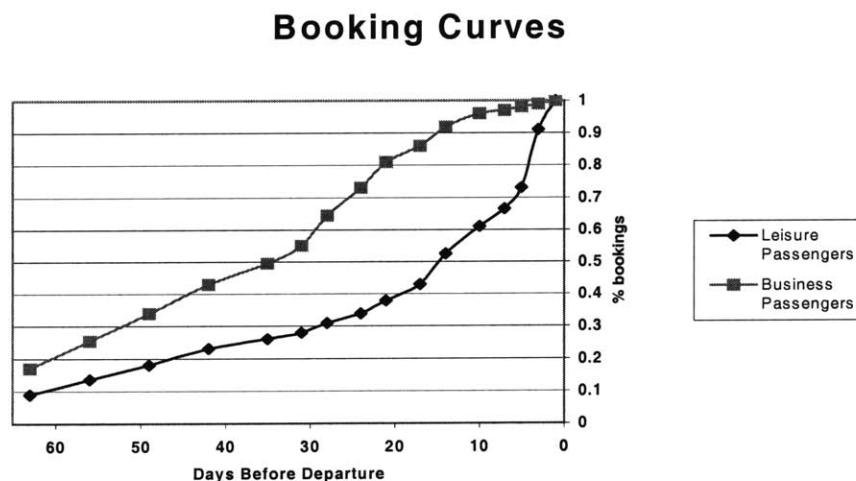
²¹ Canadian Transport Commission, *Low-Priced Air Fare Review; The First Five Years*, Report No. 1983/05E, November 1983.

It is generally agreed in the airline industry that there are two basic types of passengers that travel today:

- Business passengers who are extremely time-sensitive and do not care about how much money they will pay for a flight as they generally do not pay for the ticket themselves. In addition, these passengers have very tight time constraints and desire complete flexibility of schedule.
- Leisure passengers, who, on the other hand, are often willing to modify their plans to take advantage of cheaper flights and special fares. Unlike business passengers, they are much more sensitive to price but are willing to consider more options as far as flights are concerned and are willing to alter their plans to reduce their fares.

In order to take this into consideration in PODS, the input parameters of the simulator (namely the booking curves and the restrictions) were determined to try and match the characteristics of each passenger type. For example, each restriction is given a disutility that is a cost associated with the restriction. These costs are specific to each restriction, but also to each passenger type. Moreover, the booking patterns of each passenger type are specified as inputs to PODS, to reflect the differences between these two passenger types. These booking curves, derived from actual airline data are shown on Figure 11.

Figure 11: Booking Curves by Passenger Type



As can be seen on the previous curves, there is a major difference between leisure and business passengers in the booking pattern. Indeed, leisure passengers book early and, for example, thirty days before departure, more than fifty percent of the leisure passengers have requested their flight. However, only about thirty percent of business passengers have already made booking requests thirty days before

departure. As we get closer to departure, the business curve slope increases while the leisure curve tapers to 100%.

This first fundamental notion points out two types of passengers, among which business passengers are the more likely to sell-up to a higher class, should they be told that their first choice is unavailable or sold out.

3.1.2 Maximum Willingness-to-Pay

The notion of maximum Willingness-to-Pay (WTP) is very important to the **Belobaba-Weatherford**²² (see 3.3 below) sell-up Heuristic. Indeed, it allows the user to estimate the probability that a passenger will agree to sell-up. The maximum WTP is the maximum price a passenger is willing to pay to board a plane. The idea behind any economic model, including Revenue Management, is to get each passenger to pay his (her) maximum price. Obviously, this form of price discrimination, also known as first-degree price discrimination, is almost never achieved, and clearly not in the airline industry.

There exist two other forms of price discrimination, according to economic theory. Second-degree price discrimination occurs when the willingness to pay of a customer decreases with the amount of goods he (she) buys. Hence, this theory offers lower prices as quantity increases. Evidently, this second approach of price discrimination is quite difficult to apply in the airline industry, as a passenger rarely buys more than one ticket at a time. However, in dealing with large business companies, the airlines may decide to adopt this behavior. The third theory assumes that there are two (or more) types of customers that are willing to pay a different price for the same product.

It is not our purpose here to get into the pros and cons of each approach and what should be used by the airlines in general. However, this quick analysis of the three price-discrimination theories allows us to conclude that the model that is often used by the airlines is a mixture of first and third degree price discrimination. Indeed, airlines try to segregate demand according to willingness-to-pay, hence the business vs. leisure separation, and then price their products differently depending on differentiation in service quality.

Overall, the underlying idea in Revenue Management is to try and find the maximum WTP of each passenger and then try to have this passenger pay the fare that is closer to this WTP. Therefore, accounting for the possibility of sell-up is a way to adjust for each passenger's willingness-to-pay and hopefully increase the overall revenues of the airline.

3.2 Bohutinsky²⁰'s Study

Bohutinsky (²⁰) analyzed the effects of premature fare class closure, that is, the closure of a fare class before the final authorized booking day in this fare class. She

focused the analysis on the sell-up phenomenon and used her observations to draw very important conclusions regarding sell-up.

First of all, sell-up depends on demand: The higher the demand for a flight, the greater the probability that sell-up will occur. This result seems very logical and reasonable, and is confirmed by the fact that the higher the demand the more likely lower classes will be closed down by the Revenue Management system, and therefore the more likely passengers are to be willing to travel at a higher fare.

Second, the sell-up rate among higher fare classes appears to be higher. This result again seems very reasonable, as one would expect high yield passengers (primarily business) to have a higher willingness-to-pay.

Finally, she noted that a very competitive market may bring a totally different result than sell-up. Indeed, in that case passengers who could not be accommodated by their first choice airline could, and often would fly the competition rather than sell-up.

Regardless of the new variables such as frequent flyer programs that may have an impact on the findings of **Bohutinsky's** study²⁰, it nonetheless documents the existence of sell-up and supports the relevance of this thesis.

3.3 The Belobaba-Weatherford Heuristic²²

In 1996, Belobaba and Weatherford developed a sell-up algorithm to be used with EMSRb algorithms (therefore directly applicable to GVN and DAVN which both use EMSRb) and, later with all the Revenue Management schemes. We will briefly introduce this heuristic in the following paragraphs, but the reader is referred to **Charania**²³ for more detailed explanations regarding sell-up and all the algorithms that are currently used.

This heuristic is based upon the use of a decision tree for each passenger. At each step, the passenger decides whether he (she) will sell-up or not. To better understand the mechanisms of the heuristic, we give the example of a two-class flight. In this case, the added protection to the previously determined EMSRb protection is determined according to the following formula:

$$EMSR_b(S_2^1 + V_2^1) \times (1 - p) + p \times f_1 = f_2$$

In this formula, S_2^1 is the protection for class 1 from class 2,
 V_2^1 is the additional protection obtained through the heuristic,
 p is the probability of sell-up,
 f_1 and f_2 are the fares for classes 1 and 2.

²² Belobaba, P.P., Weatherford, L.R., *Comparing Decision Rules that Incorporate Customer Diversion in Perishable Asset Revenue Management Situations*, Decision Sciences, Volume 27, Spring 1996

²³ Charania, Aamer, *Incorporating Sell-up in Airline Revenue Management*, June 1998

This formula then leads to the more general formula below:

$$P(\pi_n) = \frac{R_{n+1} - R_{1,n} \times SU_{n+1,n}}{R_{1,n}(1 - SU_{n+1,n})}$$

Where $P(\pi_n)$ is the probability of selling the π_n th seat in class n or higher,
 π_n is the protection level for class 1 to n ,
 $R_{1,n}$ is the weighed average revenue for classes 1 to n ,
 R_{n+1} is the revenue from class immediately lower to class n ,
 $SU_{n+1,n}$ is the probability of sell-up from class $n+1$ to n .

This model was tested under two different scenarios and was shown to lead to simulated additional gains of about 2% in revenues over the basic EMSRb algorithm revenues.

A brief analysis of the heuristic shows us that the protection level depends on the fare ratio of one class to the next, the sell-up rate and the demand distribution of the higher fare class. However, it should be pointed out here that this heuristic does not take into consideration the lower-fare demand distribution. This is a striking point that comes from the fact that the algorithm does not estimate the sell-up rate itself, but only uses a value input by the user. Hence, in this algorithm, it is critical that the estimation of the sell-up rate be very good. Otherwise, the protection levels may be far from the actual number of bookings. Another aspect related to this omission is that the higher the demand, the greater the probability of spill in the lower fare classes and therefore the greater the probability of sell-up. However, this heuristic suggests that sell-up is not a function of spill, or at least that if it is, sell-up should be estimated by the airline on a flight-by-flight basis.

Nonetheless, we use this heuristic in all our analyses in this thesis, and base all our conclusions on this model, as it was shown to be beneficial on the one hand and, on the other hand, because it is a simple heuristic.

3.4 Another Sell-up Algorithm: Brumelle et al.²⁴

Brumelle et al.²⁴ developed a decision rule for a two-class example that assumes that the demands in both fare classes are not independent. The underlying idea in this study is that a portion of the lower class passengers that are denied boarding will sell-up to the higher class, thereby increasing demand for the higher class. The algorithm used to then determine booking limits is essentially the same as EMSRb other than the fact that it allows for the two demands to be dependent. Without getting into details, this algorithm then includes factors to account for the possibility of sell-up with this interdependence of demand across both classes. However, this algorithm is very difficult to use in any but a two-class case, hence making it

²⁴ Brumelle, S.L., McGill, J.I., Oum, T.H., Sawaki, K., Tretheway, M.W., *Allocation of Airline Seats between Stochastically Dependent Demands*, Transportation Science, Volume 24, 1990

difficult if not impossible to use in the “real world”. Indeed, when using more than two classes, the interdependent demands quickly make it impossible to solve the problem. A more detailed explanation of the mechanisms of the algorithm can be found in **Charania’s** work²³.

The authors provide a suggestion, as how to tackle the problem when there are more than two fare classes involved. In the end, the algorithm they come up with closely resembles the **Belobaba-Weatherford**²² heuristic.

3.5 Objectives of Sell-up

The primary objective of accounting for sell-up in Revenue Management algorithms is, as we stated earlier in this chapter, to increase revenues for the airline taking this into consideration. The questions that we will try to answer in the following simulation results are (1) how does accounting for sell-up in the Revenue Management optimization algorithm lead to revenue increases, and, (2) what are the benefits that can be gained from accounting for sell-up, depending on what the competing airline does.

To the first question, we already have an idea of the answer: if accounting for sell-up does indeed lead to revenue increases, it is by better understanding the passengers’ behavior. Indeed, as **Bohutinsky**²⁰ suggests in her research, passengers do sell-up in certain situations. Then, by using the **Belobaba-Weatherford**²² heuristic in the EMSRb algorithm, we are able to adjust our protections to account for this possibility. In all likelihood, better matching the Revenue Management algorithm to the actual passenger behavior has to bring revenue benefits.

The second question remains unanswered at this point of the thesis and must be answered through simulation, which will be the subject of the second part of the thesis. We will analyze how the revenues of the airline that is accounting for sell-up change, how they are affected by overestimating the sell-up rate, how they increase when the sell-up rate is well matched to passenger behavior, and more.

The next chapters present the simulation we ran in order to analyze the effect of sell-up with PODS and what our results showed. In particular, we look at the revenue impacts of the Belobaba-Weatherford heuristic on a Fare Class Yield management algorithm (EMSRb) and a simple O-D based algorithm (GVN), depending on the sell-up rates and detruncation method used. We show that, as expected, there are revenues to be gained from these improvements to the revenue management algorithms.

PART II: SIMULATION AND RESULTS

In Part Two of this thesis, we will present our findings regarding the influence of sell-up models and forecasting (more specifically detruncation), on the revenues of the airline. This second part of the thesis will be divided into three major chapters, Chapters Four through Six. Chapter Four will present the findings for a traditional Fare Class Yield Management (FCYM) environment, which is the still the most common in the airline industry today. Chapter Five will focus on the virtual bucket environment. At this point, we will have a good idea of the gains that can be achieved through sell-up models and detruncation modifications. Finally, in Chapter Six, we will compare these results to the gains that were previously obtained through O-D Revenue Management algorithms. In addition, we will try to validate these results in a larger network and study the influence of multiple path choices on the gains obtained by accounting for sell-up.

We find it important to add here that we recognize that we do not fully address in this thesis the question of which is the “best” input sell-up rate. We focus on the influence of sell-up models added to Revenue Management algorithms in terms of revenue gains, but, even though we look for good estimates of the sell-up rates, this is not our primary concern. In addition, as shown in Part II, Chapter Four the “best” input sell-up rates are chosen after trying a few combinations. In the “real” airline world, it would be necessary to find a way to estimate these sell-up rates and not guess what they are, as this can lead to revenue losses as shown later.

Before we start presenting our results, we would like to introduce what we will be referring to in the remainder of the thesis as the “base case”. The following paragraphs will give a brief description of the base case in addition to a few results regarding the airlines’ revenues.

Base Case

Before we can talk about the simulations themselves, we must introduce a “base case”. This “base case” will be used throughout the thesis as our point of reference and comparison. Our base case was chosen to reflect what is currently assumed to be the standard Revenue Management system used in the airline industry. It has been applied and tested by many airlines, as far as the Revenue Management method and the forecasting methods are concerned.

The simulation network itself is, as we will explain in the following paragraph, the small network 6B that has proven to have very robust simulation results.

Specifications of Our Base Case

Our base cases is set up as follows:

- Both airlines use the **leg-based** fare class, EMSRb Revenue Management algorithm. This algorithm, as we discussed earlier in this thesis, is representative of the standard Fare Class Yield Management method used in the airline industry.
- Both airlines use Booking Curve detruncation in association with Pick-up forecasting. The reader is referred to previous chapters for descriptions of both Booking Curve detruncation and Pick-up forecasting.
- The network we are running is network 6B. Briefly, Network 6B is a competitive network where each airline operates a hub operation with the same 6 spoke cities at different distances from the hubs. The network is set up to be perfectly symmetric so that neither of the airlines has a schedule-based competitive advantage. The cities lie at distances ranging from 500 to 2000 miles from the hub, which allows for O-D paths with lengths ranging from 500 to 3500 miles.

Once again, the purpose of this short chapter is to define what we will be referring to as our base case in the following chapters. Most of the results shown in the following sections will be compared to this base case, unless specifically mentioned otherwise. The base case revenues and network load factors are shown on Table 12. The reader is referred to Lee⁸ for more detailed results regarding the base case.

Table 12: Base Case Results at all Tested Demand Factors

DF	Airline A		Airline B	
	Revenue	ALF	Revenue	ALF
0.8	\$189,813.00	67.40%	\$189,703.00	67.34%
1.0	\$226,954.00	75.78%	\$226,952.00	75.75%
1.2	\$259,762.00	79.99%	\$260,029.00	80.05%

Chapter 4: Sell-up and Detruncation in a Fare Class Yield Management (FCYM) Environment

As we explained earlier, we will focus in this chapter on the impact of sell-up and detruncation on the Fare Class Yield Management environment, that is, EMSRb control. This chapter will be divided into two parts: The first part will focus on the impact of the sell-up heuristic in a “basic” detruncation environment, i.e. when the airlines are using booking curve detruncation. In addition, this first part will also allow us to set the basis for input sell-up rates, that is, the sell-up rates we estimate as the closest match to the simulated sell-up rates (that are independent of our input, in this thesis and this set of simulations). The second part will then focus on the joint effect of sell-up models and the more aggressive Projection detruncation. We will then try to explain how each change affects revenues and how.

The idea behind this first chapter is to determine the gains that can be achieved by the airline accounting for sell-up in a Fare Class Yield Management environment. Our goal here is first to assess the potential network revenue gains that can be achieved by making modifications to an existing Fare Class Yield Management optimizer.

4.1 Booking Curve Detruncation

Throughout this chapter, we focus on the gains obtained by the airlines when one airline, or both, accounts for the possibility of sell-up. The only change from the base case lies in the fact that one of the competitors, or both, has the possibility to include in its Revenue Management optimizer the **Belobaba-Weatherford**²² Sell-up heuristic. We will show in the following paragraphs that the outcome of this change is that accounting for sell-up leads to revenue gains for the airline(s) implementing the change. Moreover, we will explain the reasons for these gains.

The reader is reminded that PODS is set up in our simulations to be perfectly symmetric for both airlines (A and B), as there are no preference factors input in the simulator. Therefore, all the results that we obtain for airline A can be easily transposed to airline B (if airline B is using the same Revenue Management method, the same detruncation and forecasting method, etc.).

4.1.1 Airline A Only Accounts for Sell-up

In this case, we run simulations for several demand factors where airline A is the one accounting for sell-up and we then analyze the results. The EMSRb sell-up heuristic requires us to input the sell-up rate from one class to the next higher classes. We

must then try to evaluate as well as possible the “actual” sell-up rate. Hence, we have the possibility to input a constant sell-up rate throughout the fare classes or to input a differential sell-up rate, as suggested by Bohutinsky’s²⁰ analysis.

4.1.1.a Results

Gains as a Function of Sell-up Rate and Demand Factor

The first step in our analysis is to try various sell-up rates in the EMSRb heuristic and see what happens to revenues for both airlines, but more importantly for airline A, the airline implementing the change. Our goal in this section is to find the “best” input sell-up rates: The sell-up rates that lead to the highest revenue gains.

Constant Sell-up Rates

It turns out that revenues for airline A increase as we start inputting even low constant sell-up rates. However, as the sell-up rate increases above 30 percent, the algorithm “explodes” and leads to tremendous overprotection for high fare class seats. As the mathematical formula shows, as soon as the sell-up rate p exceeds the adjusted-fare ratio between two consecutive fare classes (the reader will recall that the protections from EMSRb are computed after taking a weighed average of each fare), the algorithm leads to infinite protection for the higher of the two classes. Therefore, the resulting protections do not make any sense. The conclusion of this first analysis of the sell-up algorithm is that the algorithm is very sensitive to demand forecasts and fare ratios. Hence, high input sell-up rates must be used with extreme caution.

Table 13: Revenue Gains as a Function of Constant Sell-up Rate and Demand Factor

DF	Sell-up Rate	Airline A			Airline B		
		Revenue	Absolute Revenue Gain over Base	Percentage Revenue Gain over Base	Revenue	Absolute Revenue Gain over Base	Percentage Revenue Gain over Base
0.8	0%	\$189,813.00	\$0.00		\$189,703.00	\$0.00	
	10%	\$190,224.00	\$411.00	0.217%	\$189,500.00	-\$203.00	-0.107%
	15%	\$190,414.00	\$601.00	0.317%	\$189,396.00	-\$307.00	-0.162%
	20%	\$190,559.00	\$746.00	0.393%	\$189,302.00	-\$401.00	-0.211%
	25%	\$190,503.00	\$690.00	0.364%	\$189,267.00	-\$436.00	-0.230%
	30%	\$178,200.00	-\$11,613.00	-6.118%	\$193,967.00	\$4,264.00	2.246%
1	0%	\$226,954.00	\$0.00		\$226,952.00	\$0.00	
	10%	\$228,028.00	\$1,074.00	0.473%	\$226,315.00	-\$637.00	-0.281%
	15%	\$228,508.00	\$1,554.00	0.685%	\$226,006.00	-\$946.00	-0.417%
	20%	\$228,846.00	\$1,892.00	0.834%	\$225,731.00	-\$1,221.00	-0.538%
	25%	\$228,807.00	\$1,853.00	0.816%	\$225,526.00	-\$1,426.00	-0.628%
	30%	\$217,067.00	-\$9,887.00	-4.356%	\$228,493.00	\$1,541.00	0.679%
1.2	0%	\$259,762.00	\$0.00	0.000%	\$260,029.00	\$0.00	
	10%	\$261,299.00	\$1,537.00	0.592%	\$259,114.00	-\$915.00	-0.352%
	15%	\$261,888.00	\$2,126.00	0.818%	\$258,718.00	-\$1,311.00	-0.504%
	20%	\$262,205.00	\$2,443.00	0.940%	\$258,423.00	-\$1,606.00	-0.618%
	25%	\$262,007.00	\$2,245.00	0.864%	\$258,281.00	-\$1,748.00	-0.672%
	30%	\$248,698.00	-\$11,064.00	-4.259%	\$260,965.00	\$936.00	0.360%

Table 13 shows that revenues gradually increase with the assumed sell-up rate for airline A, as stated earlier. Moreover, the peak revenue gain is reached at 20 percent assumed sell-up, independent of the demand factor. As soon as the input sell-up rate exceeds 20 percent, we start seeing lower revenue gains, up to the point when revenues start dropping below the base value. In particular, at 30 percent assumed sell-up, all three cases show considerable revenue losses. On the other hand, revenues increase by as much as 0.94 percent when the airline accounts for sell-up with 20 percent sell-up assumed. Moreover, as one would expect, the higher the demand factor, the larger the revenue gains, and the larger the percentage revenue gain. We will explain the reasons for this in the following chapters.

On the other hand, airline B, which is not accounting for the possibility of sell-up at this point, suffers from increasing losses as the sell-up rate assumed by airline A increases to 20 percent but, unlike airline A, its revenues continue to drop at 25 percent sell-up and only recover at 30 percent sell-up. Airline B's revenues vary with the sell-up rate assumed by airline A. In summary, airline B experiences the opposite gains and losses than airline A. However, it is interesting to note that at 25 percent assumed sell-up rate by airline A, the revenues for airline A are beginning to decrease while the revenues for B continue dropping. The reason for this is that airline A is now beginning to overprotect its high classes, while the **Belobaba-Weatherford**²² heuristic has not yet "blown up". Therefore, airline B, which is not protecting enough high class fares, ends up booking low fare leisure passengers that are spilled by A. Hence, B fills up with low fare passengers even faster than at 20 percent assumed sell-up (by airline A), and its revenues drop. At the same time, airline A is flying empty seats as it overprotected for Y and B passengers and its revenues begin decreasing.

In summary, when only airline A is accounting for sell-up, it achieves the greatest revenue gains at a constant assumed sell-up rate of 20 percent, regardless of the demand factor. Moreover, revenue gains range from 0.4% to almost 1%, depending on the demand factor.

Differential Sell-up Rate

At this point, we have tested constant sell-up rates applied to each pair of adjacent fare classes. According to **Bohutinsky's** analysis of sell-up rates²⁰ and passenger behaviors, it is more likely that business passengers have a higher probability of selling up to a higher fare class. Therefore, we need to evaluate the revenue gains at differential sell-up rates. Based on the assumption that business passengers are more likely to sell-up, we try differential sell-up rates that are assumed to be higher for sell-up from B to Y, lower from M to higher classes and even lower from Q to higher classes. We therefore tried the following input sell-up rates to the EMSRb sell-up heuristic (See Table 14):

Table 14: Differential Sell-up Rates

Case Name	Sell-up Rate		
	Q->Y,B,M	M->Y,B	B->Y
0.1-0.2-0.3	10%	20%	30%
0.2-0.3-0.4	20%	30%	40%
0.2-0.3-0.45	20%	30%	45%
0.25-0.35-0.45	25%	35%	45%
0.2-0.3-0.5	20%	30%	50%

These “differential Sell-up rates” reflect the notion that business passengers are more likely to sell up, as the sell-up rates for higher fare classes are higher. The simulation runs show that revenues attain their highest level in the third case shown on Table 14, that is for a differential set of sell-up rates of 20%, 30% and 45% per fare class, respectively for class Q, M and B. The results are shown in Table 15.

Once again, we observe exactly the same pattern as with constant assumed sell-up rates. As soon as the fare ratios exceed the input sell-up rate, the algorithm “explodes”. Moreover, as we adjust the input sell-up rates for each fare class, we increase the revenue gains for airline A, while airline B’s revenues decrease. In addition, when we exceed the “best” input sell-up rate for airline A without reaching the feasibility limit for the heuristic, airline A’s revenues begin to decrease slowly while airline B’s revenues keep dropping for the reasons explained earlier. As we increase the input sell-up rates and eventually go beyond the algorithm’s limit for airline A, the heuristic blows up and airline B captures all the passengers spilled by airline A, whose revenues are now substantially below the base case scenario.

Table 15: Revenue Gains as a Function of Differential Sell-up Rates and Demand Factor

DF	Sell-up Rate	Airline A			Airline B		
		Revenue	Absolute Revenue Gain over Base	Percentage Revenue Gain over Base	Revenue	Absolute Revenue Gain over Base	Percentage Revenue Gain over Base
0.8	0.1-0.2-0.3	\$190,582.00	\$769.00	0.405%	\$189,504.00	-\$199.00	-0.105%
	0.2-0.3-0.4	\$190,884.00	\$1,071.00	0.564%	\$189,382.00	-\$321.00	-0.169%
	0.2-0.3-0.45	\$190,948.00	\$1,135.00	0.598%	\$189,575.00	-\$128.00	-0.067%
	0.25-0.35-0.45	\$190,645.00	\$832.00	0.438%	\$189,447.00	-\$256.00	-0.135%
	0.2-0.3-0.5	\$147,846.00	-\$41,967.00	-22.110%	\$209,927.00	\$20,224.00	10.655%
1.0	0.1-0.2-0.3	\$228,895.00	\$1,941.00	0.855%	\$226,397.00	-\$555.00	-0.245%
	0.2-0.3-0.4	\$229,545.00	\$2,591.00	1.142%	\$225,998.00	-\$954.00	-0.420%
	0.2-0.3-0.45	\$229,593.00	\$2,639.00	1.163%	\$226,540.00	-\$412.00	-0.182%
	0.25-0.35-0.45	\$228,699.00	\$1,745.00	0.769%	\$226,119.00	-\$833.00	-0.367%
	0.2-0.3-0.5	\$187,583.00	-\$39,371.00	-17.348%	\$244,014.00	\$17,062.00	7.518%
1.2	0.1-0.2-0.3	\$262,878.00	\$3,116.00	1.200%	\$259,185.00	-\$844.00	-0.325%
	0.2-0.3-0.4	\$263,505.00	\$3,743.00	1.441%	\$258,921.00	-\$1,108.00	-0.426%
	0.2-0.3-0.45	\$263,596.00	\$3,834.00	1.476%	\$259,825.00	-\$204.00	-0.078%
	0.25-0.35-0.45	\$262,208.00	\$2,446.00	0.942%	\$259,453.00	-\$576.00	-0.222%
	0.2-0.3-0.5	\$227,747.00	-\$32,015.00	-12.325%	\$274,093.00	\$14,064.00	5.409%

In summary, the highest revenue gain is achieved for airline A at input differential sell-up rates of 20%, 30% and 45% per fare class, regardless of the demand factor. Percentage revenue gains range from 0.6% to 1.5% depending on the demand factor.

4.1.1.b Conclusion

Overall, this first analysis enabled us to determine the “best” sell-up rate in the case when only one of the two competitors accounts for the possibility of sell-up, in a Fare Class Yield Management environment, for our particular simulation environment. The “best” input sell-up rates for the EMSRb heuristic happen to be 20 percent per fare class in the constant sell-up rate case, and 20%, 30% and 45% per fare class in the differential sell-up rate case. Once again, these sell-up rates were determined by trying a few combinations and may not be the actual absolute best sell-up rates. They are an estimate of the actual sell-up rates that the airlines would also try to estimate in order to incorporate sell-up models in their Revenue Management system.

Moreover, in this first part, we explained briefly what happens when the input sell-up rates become too high and the algorithm starts overprotecting and spilling passengers towards the competing airline. We distinguished between the case when airline A overprotects and when the heuristic “blows up”. This points out an intuitive but very important result: too much sell-up assumed can lead revenue losses.

We now return in more detail to what happens when one airline accounts for sell-up.

4.1.2 Understanding What Happens

In this section we will explain what happens to demand, how passengers book their flight, and try and understand why accounting for sell-up brings revenue gains. In particular, we will be looking at fare class loads on each leg and comparing them to the base-case loads and to the competitor’s loads.

4.1.2.a The Heuristic

First of all, it is quite obvious that when we input a sell-up rate, the Belobaba-Weatherford heuristic increases the protection for the affected fare classes. In particular, as the airplane’s capacity does not vary, the protection increases for the upper classes and decreases for the lower classes. The following example shows what happens to protections when we account for sell-up in a very specific case that is only meant to illustrate the increase in protections.

Table 16: Influence of Sell-up Heuristic on Protections

Fare Class	Fare	Avg Demand	Std Dev	Avg Overall Demand	Std Dev	Without Sell-up			With Sell-up			
						Weighted Fare Ratio	Top Down Protection	Nested Protection	Sell-up Rate	Fare Ratio	Top Down Protection	Nested Protection
Y	\$ 800.00	25	10	25	10.00	0.750	18.26	100.00	0.2	0.688	20.11	100.00
B	\$ 400.00	25	10	50	14.14	0.833	36.32	81.74	0.2	0.792	38.51	79.89
M	\$ 300.00	25	10	75	17.32	0.850	57.05	45.43	0.2	0.813	59.63	41.37
Q	\$ 200.00	25	10	100	20.00			-	0.2			-

We see here that when we use a constant sell-up rate of 20 percent per fare class, the protection increases by roughly two seats in all fare classes. Therefore, fewer seats are available for low fare passengers and more will then be turned down.

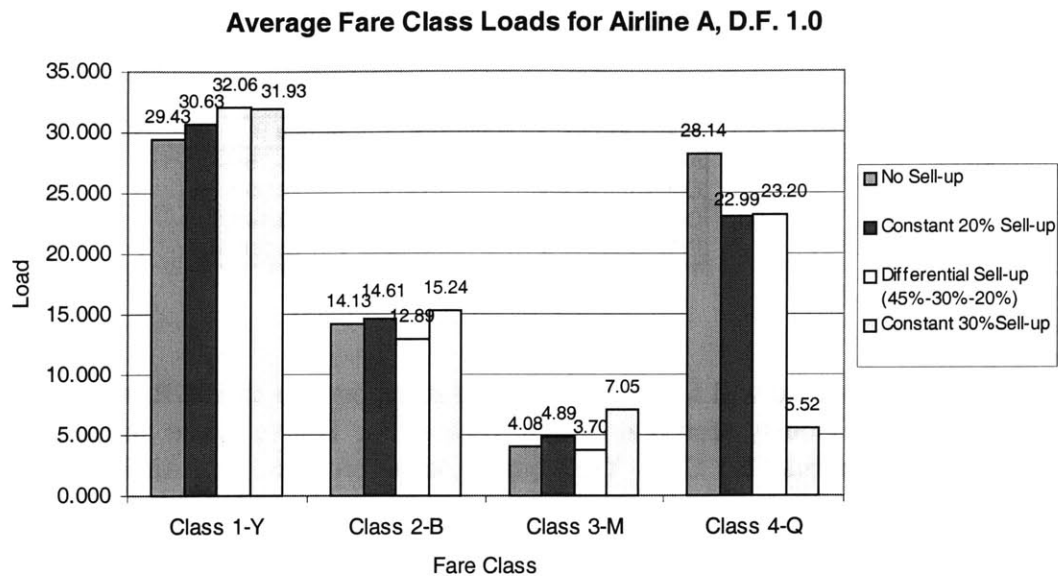
4.1.2.b Loads

Second, in terms of average leg load factors and average fare class loads, we then note that when airline A accounts for sell-up, its average leg load factors decrease compared to when it did not account for sell-up on the one hand and are lower than airline B's average leg load factors on the other hand (c.f. Table 17). In this section, we will see how the sell-up rates affect loads, both for the airline accounting for sell-up and for the airline not accounting for sell-up. The result of this analysis will show that airline A, which is accounting for sell-up, benefits from higher loads in Y, B and M class and lower loads in Q class but lower overall loads, increasing its revenues when it is not spilling too many passengers.

Airline A

Figure 12 shows, at demand factor 1.0, the average leg loads by fare class depending on the input sell-up rate, for airline A.

Figure 12: Fare Class Loads as a Function of Input Sell-up Rate, DF 1.0



It clearly appears on this graph that airline A benefits in terms of Y passengers when it is accounting for sell-up. In particular, the more sophisticated the sell-up rate gets (and hence the closer it gets to replicating actual passenger behavior), the more Y passengers book the flights, on average. This is true even when our input sell-up rates are too high. Similarly, the amount of B and M class passengers also increase with the input sell-up rates, as more low fare Q class passengers are turned down when they want to book a flight.

Q class passengers therefore decrease in number as the assumed sell-up rate increases and eventually drop very low as shown by the last bar on the graph. This represents what happens when we overestimate the “actual” sell-up rate. The loads in fare classes Y, B and M are high compared to the other three cases shown here. However, the tremendous loss in Q class passengers cannot be compensated by these higher loads in the first three fare classes. Overall, the airline then loses revenues.

It is interesting to note here that constant input sell-up rates lead to higher loads in B and M classes than differential input sell-up rates. However, in Y class, there are higher loads when we input differential sell-up rates than when we assume constant sell-up rates, and the difference is such that it outweighs the previous load difference, in terms of revenues. Therefore, regardless of the higher loads in B and M classes at constant assumed sell-up rates for airline A, the revenues are higher at differential assumed sell-up rates.

Overall, we see that the loads for airline A change in nature as the assumed sell-up rates vary. The “better” the assumed sell-up rates the more high fare passengers we get. Moreover, in terms of average leg load factor, we note that the average leg load

factor tends to decrease as the input sell-up rate increases for airline A, as shown in Table 17.

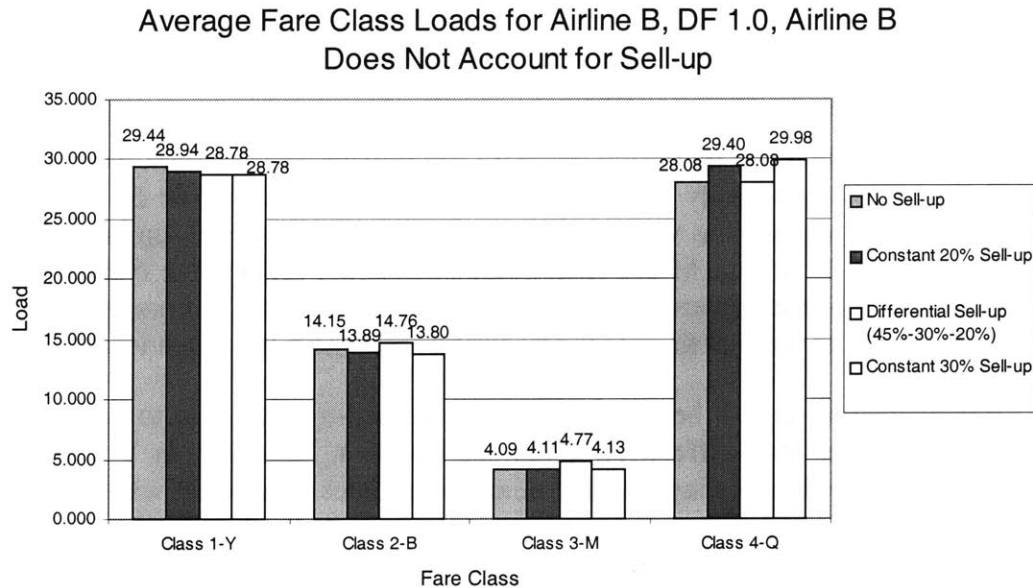
Table 17: Average Leg Load Factors, D.F. 1.0

Sell-up Rate	Average Leg Load Factor	
	Airline A	Airline B
No Sell-up	75.78%	75.75%
Constant 20%	73.12%	76.33%
Constant 30%	59.73%	80.14%
Differential	71.85%	76.39%

Airline B

In this section, we will focus briefly on what happens to airline B in terms of loads. We have previously seen that revenues for airline B drop when airline A accounts for sell-up in this Fare Class Yield Management environment. Airline B's revenues drop by as much as 0.6 percent. However, as can be seen in Table 13 and Table 15, revenues for airline B are higher when airline A uses our "best" differential input sell-up rates than when airline A uses the "best" input constant sell-up rates. Airline A has higher gains with differential sell-up rates assumed and airline B has lower losses. This is an unexpected result that we will explain in the following paragraphs.

Figure 13: Average Fare Class Loads for Airline B, DF 1.0



As shown in Figure 13, as the input sell-up rates increase for airline A, the loads in Y class decrease for airline B, as compared to our base case. This, along with a similar decrease in loads in B class (for all cases but the differential sell-up rate case, as shown in Figure 13), is the major cause of revenue decrease for airline B. Along with this reduction in the number of Y and B class passengers, we note an increase in the

number of M and Q class passengers. This can easily be explained by the fact that airline A now spills more of these passengers. Airline B's Revenue Management algorithm protects fewer seats for higher yield passengers and therefore allows some of these spilled passengers to fill up its lower classes and, in turn to eat up some space that could have been saved for Y or B class passengers. Therefore, airline B books more M and Q class passengers as airline A begins accounting for sell-up.

As for the differential sell-up rate case, we note that the loads for airline B are higher in B and M class than the base case and all of the other input sell-up cases. This explains that the revenues for airline B are higher in this case than in the other sell-up cases and explains the result that we pointed out earlier. At the same time, we also pointed out that the revenues for airline A in this case are higher than at any other constant input sell-up rate because we better model the consumers' behavior.

Overall, in this situation, both airlines benefit from Airline A switching to differential input sell-up rates as opposed to constant sell-up rates assumed. Finally, in terms of average leg load factors, Table 17 shows that, as we expected, average loads increase for airline B as the input sell-up rates used by airline A increases and as average leg loads decrease for airline A, which spills more passengers.

4.1.2.c Summary

In summary, we note in this first set of results that when only one airline is accounting for sell-up, it undoubtedly benefits from this additional understanding and modeling of passenger behavior, provided it correctly chooses the input sell-up rates to EMSRb. In particular, we conclude that in the case where only one airline is accounting for sell-up, the best input sell-up rate tested is a differential rate of 45%, 30% and 20% by fare class, from top down. The revenue gains for airline A ranged from 0.4% to 1% in the case of the 20% assumed sell-up and from 0.6% to 1.5% in the case of differential sell-up rates assumed, depending on demand factors.

Another interesting result that we noted here is that the revenue losses for the competitor (who is not accounting for sell-up) are less when airline A uses differential sell-up rates and maximizes its revenues. Indeed, airline B loses between 0.2% and 0.6% revenues depending on demand factor at a constant input sell-up rate by airline A while it only loses between 0.1% and 0.2% when airline A uses differential sell-up rates assumed.

Once again, what we focus on here is the possible benefits from accounting for sell-up. However, we do recognize that the benefits are highly dependent on the choice of the sell-up rates, but we do not focus, in this thesis, on how to evaluate the true sell-up rates. Rather, we try and get a feel for the possible revenue gains, should the airlines be able to evaluate the sell-up rates properly and use them in a sell-up model.

4.1.3 Both Airlines Account for Sell-up

In this section, we now focus on what happens when both airlines account for sell-up through the EMSRb heuristic. Indeed, now that we know that airline A can improve its revenues from our base case when it accounts for sell-up, we would like to know whether these results still hold when the competitor upgrades its Revenue Management algorithm to account for sell-up. Our expectation, which will prove to be true, is that both airlines should still gain from accounting for sell-up, but to a lesser degree than when only airline A was accounting for sell-up.

4.1.3.a Results

We once again used PODS to run different types of simulations. As in the previous section, we ran both constant and differential assumed sell-up rates. We will follow the same structure in this section as previously: first we will examine the entire set of results to get an idea of the magnitude of the possible gains and evaluate the “best” input sell-up rates. We will then explain these results.

Constant Sell-up Rates

At constant input sell-up rates used by both airlines, we see from Table 18 that airline A and B both now achieve revenue gains over our base case. In particular, we note that the highest revenue gains are now achieved at 15% sell-up rate assumed for all fare classes. This makes sense as both airlines are now protecting more while demand remains the same. They therefore split high revenue passengers evenly and must protect fewer seats in higher fare classes. This then leads to a lower “optimal” input sell-up rate.

Table 18: Revenue Gains as a Function of (Constant) Input Sell-up Rate and Demand Factor

DF	Sell-up Rate	Airline A			Airline B		
		Revenue	Absolute Revenue Gain over Base	Percentage Revenue Gain over Base	Revenue	Absolute Revenue Gain over Base	Percentage Revenue Gain over Base
0.8	0%	\$189,813.00			\$189,703.00		
	10%	\$190,012.00	\$199.00	0.105%	\$189,885.00	\$182.00	0.096%
	15%	\$190,062.00	\$249.00	0.131%	\$189,940.00	\$237.00	0.125%
	20%	\$190,046.00	\$233.00	0.123%	\$189,929.00	\$226.00	0.119%
	25%	\$189,845.00	\$32.00	0.017%	\$189,694.00	-\$9.00	-0.005%
	30%	\$186,652.00	-\$3,161.00	-1.665%	\$187,038.00	-\$2,665.00	-1.405%
1.0	0%	\$226,954.00			\$226,952.00		
	10%	\$227,360.00	\$406.00	0.179%	\$227,345.00	\$393.00	0.173%
	15%	\$227,440.00	\$486.00	0.214%	\$227,415.00	\$463.00	0.204%
	20%	\$227,347.00	\$393.00	0.173%	\$227,340.00	\$388.00	0.171%
	25%	\$226,890.00	-\$64.00	-0.028%	\$226,869.00	-\$83.00	-0.037%
	30%	\$223,696.00	-\$3,258.00	-1.436%	\$223,441.00	-\$3,511.00	-1.547%
1.2	0%	\$259,762.00			\$260,029.00		
	10%	\$260,328.00	\$566.00	0.218%	\$260,617.00	\$588.00	0.226%
	15%	\$260,427.00	\$665.00	0.256%	\$260,722.00	\$693.00	0.267%
	20%	\$260,326.00	\$564.00	0.217%	\$260,610.00	\$581.00	0.223%
	25%	\$259,680.00	-\$82.00	-0.032%	\$260,085.00	\$56.00	0.022%
	30%	\$255,444.00	-\$4,318.00	-1.662%	\$253,378.00	-\$6,651.00	-2.558%

Another interesting point here is that the revenue gains have now dropped from what we previously observed when only airline A accounted for sell-up. We now get revenue gains between 0.1% and 0.3% depending on the demand factor (compared to 0.4% to 1%).

Overall, at constant input sell-up rates, when both airlines account for sell-up, the greatest revenue gain is achieved at a lower input sell-up rate than when only one airline was accounting for sell-up. In addition, the percentage revenue gain is smaller. However, the important point of this analysis is that in this case, both airlines are now better off than when neither airline was accounting for sell-up.

Differential Sell-up Rates

In this section, we once again focus on the influence of differential sell-up rates assumed on revenue gains, or losses, for both airlines. Once again, we use **Bohutinsky's** analysis to justify our choice of differential sell-up rates assumed as a potentially revenue generating approach to modifying the EMSRb algorithm to account for sell-up. Once again, we try various input sell-up rates (in the heuristic) and compare the performance of both airlines under these assumed sell-up conditions in terms of revenues to our base case. The results of our simulation are shown in Table 19. The notation for each case is the same as the one described in the previous section.

Table 19: Revenue Gains as a Function of (Differential) Input Sell-up Rate and Demand Factor

DF	Sell-up Rate	Airline A			Airline B		
		Revenue	Absolute Revenue Gain over Base	Percentage Revenue Gain over Base	Revenue	Absolute Revenue Gain over Base	Percentage Revenue Gain over Base
0.8	0.1-0.2-0.3	\$190,333.00	\$520.00	0.274%	\$190,232.00	\$529.00	0.279%
	0.2-0.3-0.4	\$190,362.00	\$549.00	0.289%	\$190,258.00	\$555.00	0.293%
	0.2-0.3-0.45	\$190,598.00	\$785.00	0.414%	\$190,476.00	\$773.00	0.407%
	0.2-0.3-0.5	\$167,631.00	-\$22,182.00	-11.686%	\$163,773.00	-\$25,930.00	-13.669%
1.0	0.1-0.2-0.3	\$228,165.00	\$1,211.00	0.534%	\$228,209.00	\$1,257.00	0.554%
	0.2-0.3-0.4	\$228,058.00	\$1,104.00	0.486%	\$228,104.00	\$1,152.00	0.508%
	0.2-0.3-0.45	\$228,592.00	\$1,638.00	0.722%	\$228,613.00	\$1,661.00	0.732%
	0.2-0.3-0.5	\$211,300.00	-\$15,654.00	-6.897%	\$202,613.00	-\$24,339.00	-10.724%
1.2	0.1-0.2-0.3	\$261,773.00	\$2,011.00	0.774%	\$262,016.00	\$1,987.00	0.764%
	0.2-0.3-0.4	\$261,602.00	\$1,840.00	0.708%	\$261,933.00	\$1,904.00	0.732%
	0.2-0.3-0.45	\$262,584.00	\$2,822.00	1.086%	\$262,942.00	\$2,913.00	1.120%
	0.2-0.3-0.5	\$248,038.00	-\$11,724.00	-4.513%	\$243,591.00	-\$16,438.00	-6.322%

In this case, we see that the greatest gain is reached at the same input sell-up rates as previously, that is, at 45%, 30% and 20% assumed sell-up rate per fare class, from top down at all demand factors. This tends to indicate that this combination of input sell-up rates matches rather well the actual behavior of passengers.

The first obvious observation is that we now achieve lower revenue gains compared to our previous case when only airline A was accounting for sell-up. Indeed, revenue gains have now dropped to 0.4% to 1.1% from 0.6% to 1.5% depending on demand factor.

Once again, this result was to be expected as both airlines become more aggressive in protecting seats for high yield passengers and therefore have to share the additional sell-up revenues evenly between them, which was not the case when only airline A was accounting for sell-up.

It is also interesting to note here that the gap between the gains achieved with constant input sell-up rates and differential input sell-up rates has now increased as compared to when only airline A was accounting for sell-up. Switching to differential sell-up rates assumed now increases gains by as much as 0.8% while the difference was about 0.4% when only airline A was accounting for sell-up. Overall, we see that differential sell-up rates assumed are clearly the more beneficial way for the airlines to be accounting for sell-up when both airlines do so, in terms of revenue generation.

4.1.3.b Explaining the Results

In this section, we will be looking at loads to explain the revenue gains for both airlines. At this point, given our previous analysis of loads when only airline A accounts for sell-up, we expect to see similar trends. As for the booking limits and

the effect of the heuristic on protections, the reader is referred to Part I, Chapter 3.3 above for more detail.

Comparison with Base Case Loads

Let us first of all see how the loads for airline A and B (they are very similar, as both airlines account for sell-up) change from base and as a function of the input sell-up rates.

Figure 14: Average Fare Class Loads, DF 1.0

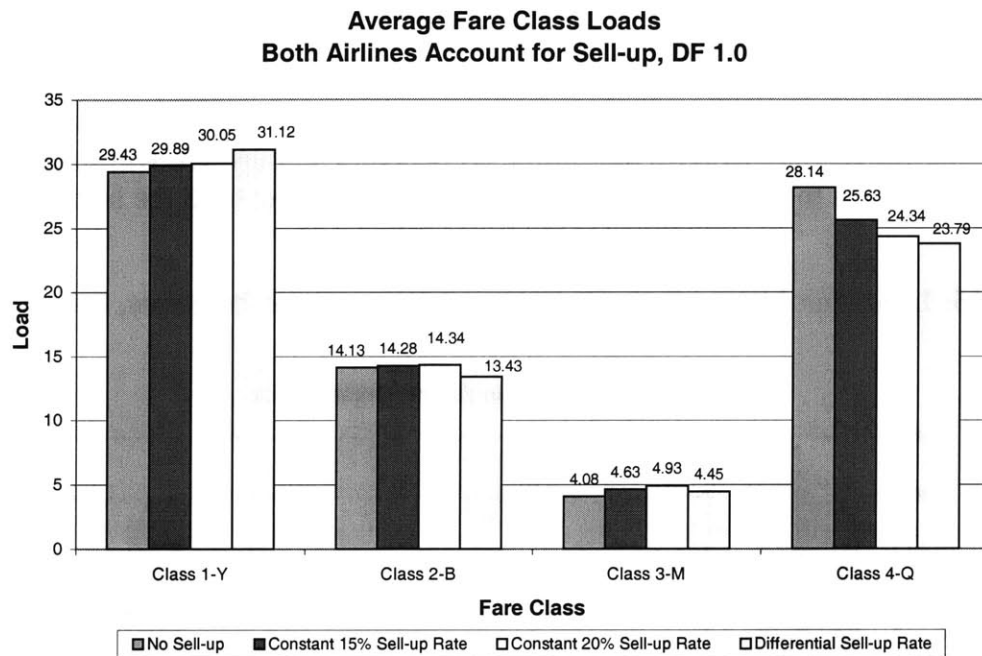


Figure 14 clearly shows what we expected: Differential input sell-up rates allow for more Y-class passengers to fly, on average, thus increasing revenues. At the same time, these differential sell-up rates reduce the protection in lower fare classes. Therefore, we see that the differential sell-up case has lower bookings, on average, in B, M and Q classes. This difference is clearly compensated by the gain in Y class passengers.

Figure 14 also enables us to explain why the revenues at 20% input sell-up rates are lower than at 15% input sell-up rates. Indeed, the graph shows that the loads in Y, B and M class are higher for 20% sell-up rate assumed. However, the loads in Q class are substantially lower at this higher sell-up rate, and this is not compensated by the slightly higher loads in all three higher fare classes. Overall, the airlines have lower revenue gains at these higher input sell-up rates.

Finally, in terms of average leg load factors, we note that as the input sell-up rates generate more revenues, the average load factor decreases and that both airlines have essentially the same load factors (for symmetry reasons, as shown in Table 20).

These average loads are higher than when only airline A was accounting for sell-up, at differential sell-up rates assumed, as both airlines now protect more for the higher fare classes and force more passengers to sell up.

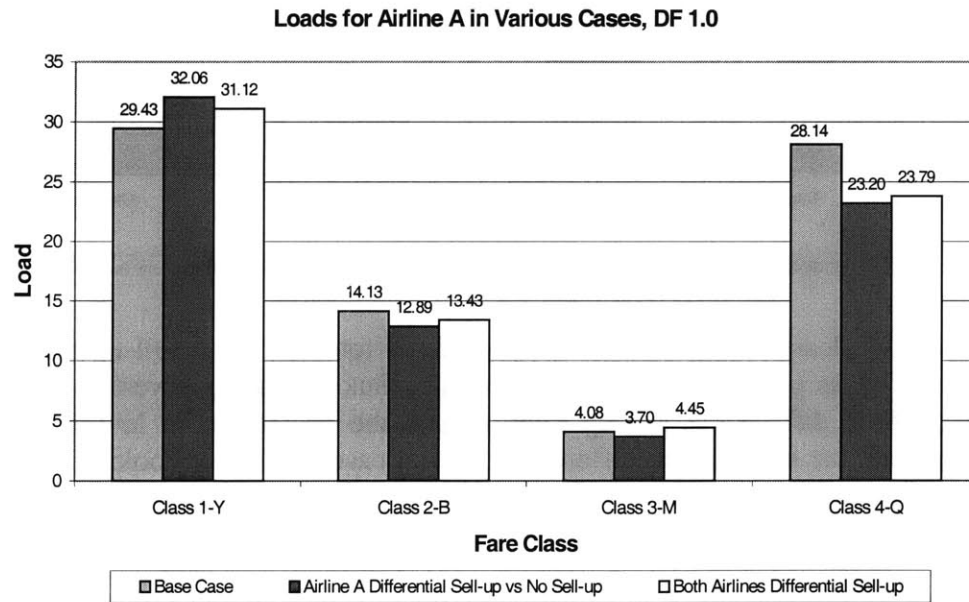
Table 20: Average Leg Load Factors, DF 1.0

Sell-up Rate	Average Leg Load Factor	
	Airline A	Airline B
No Sell-up	75.78%	75.75%
Constant 15%	74.44%	74.43%
Constant 20%	73.66%	73.63%
Differential	72.79%	72.71%

Comparison with Loads when Only Airline A Accounts for Sell-up

In this section, we focus on the differences between the case when both airlines account for sell-up and only one airline accounts for sell-up. We will therefore use the load charts to determine how loads change from one case to the next for airline A.

Figure 15: Loads for Airline A as a Function of Competitive Response, DF 1.0



We see on this chart that when only airline A is accounting for sell-up, its loads are the highest in Y class, but the lowest of all three cases in the other three classes. Yet, airline A then has the highest absolute revenues of all. This is due to the fact that the loads are high in Y class and comparatively not “too” low in the other three fare classes.

When both airlines are accounting for sell-up, the loads for airline A (and B) are high in Y, B and M classes and lower in Q class. This leads to higher revenues than

the base case and yet lower than in the case when only airline A accounts for sell-up. Indeed, both airlines now have to compete equally for high yield passengers. Therefore, loads are higher in the intermediate classes and slightly lower in Y class than when only airline A was accounting for sell-up. However, all these loads (Y, B and M) are higher than for the base case. Overall, this leads to the situation we have been describing earlier.

Finally, we observe here that the major difference in both cases with the base case is that the loads in Y class are higher.

In terms of average leg load factors, we show in Table 21 that the average leg load factor increases for airline A when both airlines account for sell-up and assume differential sell-up rates (compared to when only airline A was accounting for sell-up). This can be explained by the fact that both airlines are now protecting more in the higher classes and therefore forcing more passengers to sell up. For airline B, however, the average leg load factor decreases, which again is natural, as airline B is now protecting more for higher fare passengers and spilling more low fare passengers.

Table 21: Comparison in Average Leg Load Factors, DF 1.0

	Average Leg Load Factor			
	Airline A with Sell-up vs. B no Sell-up		Airline A with Sell-up vs. B with Sell-up	
	Airline A	Airline B	Airline A	Airline B
No Sell-up	75.78%	75.75%	75.78%	75.75%
Differential	71.85%	76.39%	72.79%	72.71%

4.1.4 Conclusion

We conclude from this first set of results that there is clearly revenue to be gained from accounting for sell-up whether the competitor does the same or not. We have seen that revenue gains range from 0.4% to 1.5% depending on the assumed sell-up rate and the competitive situation. In particular, gains are higher when only one airline accounts for sell-up, as it is then able to attract more of the high fare, high revenue Y class passengers. However, we also established in these results that the choice of the assumed sell-up rates is critical: If the assumed sell-up rates are too high, the airline overprotects and suffers very important losses. If the assumed sell-up rates are too low, the airline does not get all the benefit it should.

The next question we will now focus on in the remainder of this chapter is what happens when we change the forecasting, and more specifically the detruncation method, as this represents another approach to increasing protection levels.

4.2 Projection Detruncation

In this section, we focus on what happens when one of the airlines (or both) switches to the more aggressive Projection detruncation described in Part I, Chapter 2.1.2. This can lead to increased protection as we briefly described in the first part of the thesis. Indeed, as the τ value varies we are able to increase or decrease the estimation of the unconstrained demand and therefore to adjust the forecast and in turn the protection levels.

The goal of this section is to see if there is any additional gain that can be achieved by switching detruncation models in addition to the gains we already achieved by accounting for sell-up. In a first step, we will try and find the best τ value for Projection detruncation. By this we mean search for the τ value that gives the best results (the highest revenues) and at the same time compare this result to the previously observed revenues obtained with Booking Curve detruncation.

Our base value for τ is 0.15, as described in Part I, Chapter 2.1.2. This is the value we start from and evolve from there, depending on the results we get.

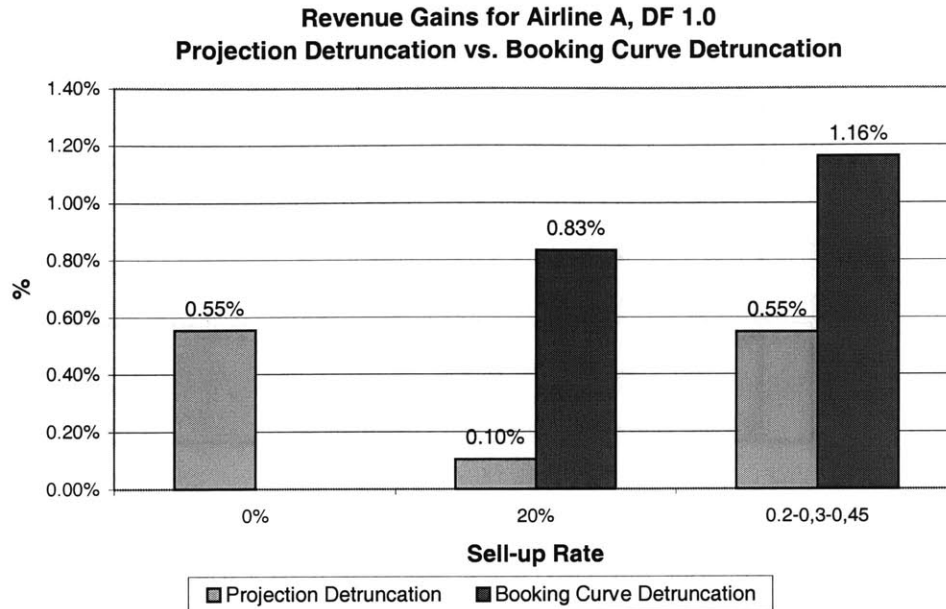
4.2.1 Preliminary Results

In this section, we re-run the previous PODS simulation cases with the “base” τ value. In the following paragraphs, we will be using either a constant input sell-up rate of 20% by fare class or a differential input sell-up rate of 45%, 30% and 20% per fare class combined with Projection detruncation at $\tau=0.15$. These assumed sell-up rates have consistently proven to give good results and we will therefore focus on these particular rates.

4.2.1.a Only Airline A Accounts for Sell-up

First of all, we are interested in what happens to revenues when we switch to Projection detruncation with the base value for τ , when only airline A is accounting for sell-up. It turns out from our results that airline A still has revenue gains when compared to our base case. However, these gains are lower than they were with Booking Curve detruncation. This is shown on Figure 16.

Figure 16: Revenue Gains as a Function of Detruncation and Sell-up, DF 1.0



We clearly see in Figure 16 that more aggressive detruncation is beneficial to the airline, when no sell-up rate is input. However, as soon as sell-up rates are added, the revenue gains drop dramatically. The reason for this is that sell-up and detruncation have essentially the same effect on the Revenue Management algorithm: They increase the protection in one way or another. Projection detruncation with a low τ value works towards increasing the forecast by increasing the estimate of the unconstrained demand while accounting for sell-up increases the protection by taking into consideration the probability that a passenger will be willing to pay more. Therefore, in the end, we end up overprotecting through the addition of both effects.

This first set of results shows us this overprotection:

- Projection detruncation can bring revenue gains, as shown at 0% assumed sell-up rates, and,
- Aggressive Projection detruncation combined with the assumed sell-up rates leads to overprotection and lower revenue gains.

The same trends are observed at other demand factors. That is, there is a definite revenue gain over the base case when airline A does not account for sell-up. However, as soon as airline A begins to account for sell-up, revenue gains are lower when using projection detruncation rather than Booking Curve detruncation. This is summarized in Table 22. Overall, however, there is a decrease in revenues by as much as 0.45% at demand factor 1.2 compared to the more beneficial Booking curve detruncation - Differential sell-up rates combination.

Table 22: Summary of Revenue Gains with Projection Detruncation, $\tau=0.15$

DF	Case	Detruncation	Sell-up	Percentage Revenue Gain (Airline A)
0.8	Base	BC	0%	
	0	Proj (Tau=0.15)	0%	0.15%
	20	Proj (Tau=0.15)	20%	0.36%
	0.2-0,3-0,45	Proj (Tau=0.15)	Differential	0.52%
1.0	Base	BC	0%	
	0	Proj (Tau=0.15)	0%	0.55%
	20	Proj (Tau=0.15)	20%	0.79%
	0.2-0,3-0,45	Proj (Tau=0.15)	Differential	1.05%
1.2	Base	BC	0%	
	0	Proj (Tau=0.15)	0%	0.59%
	20	Proj (Tau=0.15)	20%	0.76%
	0.2-0,3-0,45	Proj (Tau=0.15)	Differential	1.06%

4.2.1.b Both Airlines Account for Sell-up

We now focus on what happens when we switch to Projection detruncation for airline A only, but when both airlines account for sell-up. In the previous sections, we showed that accounting for sell-up increases the revenues of the airline implementing the change over our base case. However, when both airlines account for sell-up, the revenues (for airline A) are lower than when only one of the two airlines accounts for sell-up. The goal of this section is to study what happens when airline A switches to the more aggressive Projection detruncation (at $\tau=0.15$), when both airlines account for sell-up. Does airline A increase its revenues? Does airline B increase its revenues, or do the gains remain the same as when both airlines were using Booking curve detruncation?

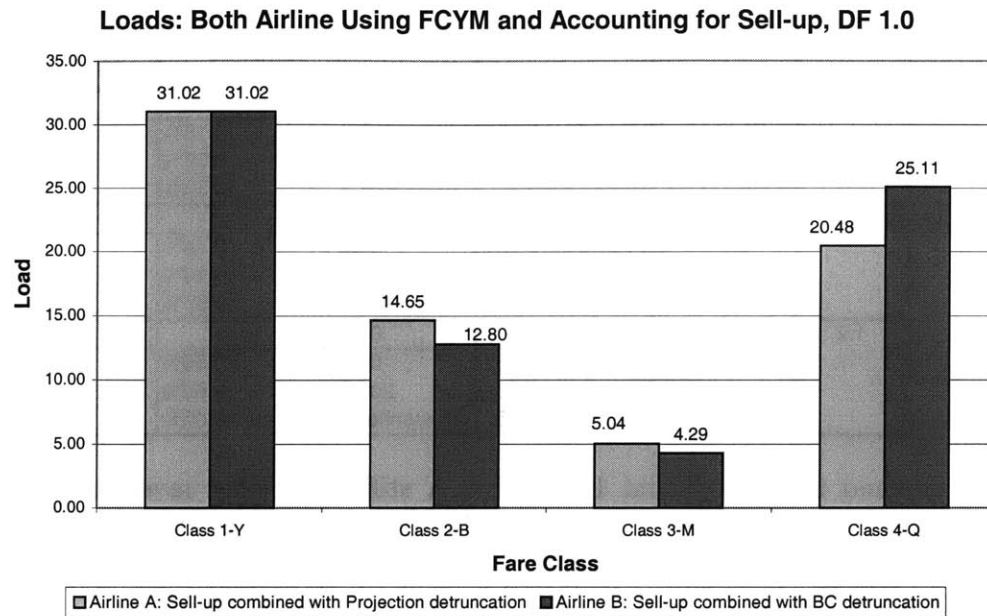
In Table 23, we show the percentage revenue gains (losses) of airlines A and B when they both account for sell-up, while only airline A uses Projection detruncation. It is clear that airline A still benefits from the more aggressive Projection detruncation when neither airline accounts for sell-up. Airline B, on the other hand suffers from its less aggressive detruncation and has revenue losses (compared to our base case). However, as soon as we input a constant or differential sell-up rate for both airlines, we see that the results become very different: It becomes unclear which airline is better off.

Table 23: Percentage Revenue Gains When Both Airlines Account for Sell-up and A Only Uses Projection Detruncation ($\tau=0.15$)

DF	Case	Detruncation	Sell-up	Percentage Revenue Gain (Airline A)	Percentage Revenue Gain (Airline B)
0.8	Base	BC	0%		
	0	Proj (Tau=0.15)	0%	0.15%	-0.15%
	20	Proj (Tau=0.15)	20%	0.07%	0.05%
	0.2-0,3-0,45	Proj (Tau=0.15)	Differential	0.32%	0.27%
1.0	Base	BC	0%		
	0	Proj (Tau=0.15)	0%	0.55%	-0.40%
	20	Proj (Tau=0.15)	20%	0.10%	0.08%
	0.2-0,3-0,45	Proj (Tau=0.15)	Differential	0.55%	0.45%
1.2	Base	BC	0%		
	0	Proj (Tau=0.15)	0%	0.59%	-0.34%
	20	Proj (Tau=0.15)	20%	0.00%	0.28%
	0.2-0,3-0,45	Proj (Tau=0.15)	Differential	0.55%	1.11%

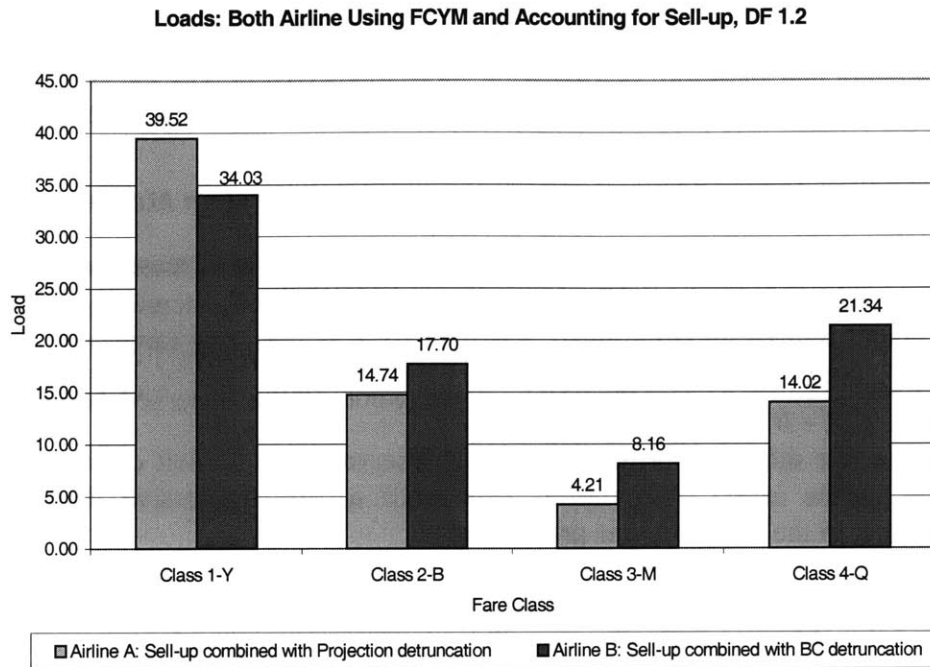
At demand factors 0.8 and 1.0, airline A still has higher revenue gains than B. Moreover, both airlines are still above the initial base case revenues. However, it is interesting to note here that the revenues are lower than they were when both airlines were using Booking Curve detruncation (and accounting for sell-up). Indeed, as the reader will recall, the revenue gains reached 0.72% at demand factor 1.0 with differential sell-up rates while we are now at best at 0.55% for the airline using Projection detruncation. This can be explained by the fact that, now, both airlines turn down initial low fare passengers, but in greater numbers. Some of these passengers fill up part of airline B's capacity, thus reducing its overall revenue gains. At the same time, airline A captures more of the high yield passengers in the last time frames before departure. However, these passengers cannot make up for the low fare passengers spilled in the early time frames, as these passengers are shared between airline A and B. Figure 17 shows the loads at demand factor 1.0, which confirms our explanation: Airline A has higher loads in the first three fare buckets while it has substantially lower loads in Q class.

Figure 17: Loads for both airlines with differential sell-up rates and demand factor 1.0



At demand factor 1.2, we observe revenue gains (for airline A) that are lower than they used to be when neither airline was accounting for sell-up. Moreover, it is now the airline that is using Booking Curve detruncation that achieves the highest gains. This can again be explained by the fact that, at high demand factor, airline A, which is overprotecting for the highest class, ends up rejecting too many lower fare passengers (B, M and Q). Overall, even though the loads are much higher in Y-class for airline A (c.f. Figure 18) the difference in loads in the other three fare classes leads to lower revenues for airline A.

Figure 18: Loads for both airlines at differential sell-up rates and demand factor 1.2



Finally, Figure 18 shows us that the loads in the three lower fare classes are much higher for airline B, which outweighs the difference in Y-class, as we explained earlier and leads to higher revenue gains for airline B at demand factor 1.2 when both airlines are accounting for sell-up while only airline A is using Projection detruncation.

4.2.1.c Summary

We have seen in the two previous sections that, first of all, Projection detruncation (at $\tau=0.15$) combined with sell-up leads to overprotection. Indeed, this leads to lower revenue gains for the airline using Projection detruncation. Second, we also observed that, when opposed to an airline also accounting for sell-up, Projection detruncation does worse than Booking Curve detruncation. Indeed, the load graphs showed us the overprotection that results from the use of the more aggressive Projection detruncation algorithm.

In the following sections, we will now see if we can improve the revenues by changing the τ value. That is, we will be testing whether there may be a slightly more aggressive detruncation method than Booking Curve detruncation that may allow for higher revenues than those achieved through Booking Curve detruncation.

4.2.2 Best τ Value and Related Revenues

In the following sections, we modify the τ value in order to evaluate whether there is additional gain (over gains previously obtained by accounting for sell-up) to be

achieved by increasing the detruncation. In all the cases described, a good indicator of the aggressiveness of the detruncation model is the “no sell-up” case. Indeed, in this case, if the airline using Projection detruncation has positive revenue gains, it implies, as confirmed by the load charts, that it is protecting more for higher fare passengers and therefore detruncating more aggressively. On the other hand, if we see revenue losses, then the airline is effectively not detruncating enough.

4.2.2.a Decreasing the Protection through the Detruncation Method

In this paragraph, we relate the results obtained when we increase the τ value, which corresponds to using less aggressive detruncation. Indeed, increasing the τ value, as the reader will recall from Part I, Chapter 2.1.2, is basically increasing the value of the ratio $\frac{B}{A+B} = \tau$, and therefore, as the value of the observed bookings has not changed, we move the projection of this observation to the left of the curve, that is, closer to the initial observation. This leads to less aggressive detruncation and therefore, in the end, to lower protections.

Therefore, in this paragraph, we describe and explain our results when we modify the τ value. More specifically, we will be running two types of cases:

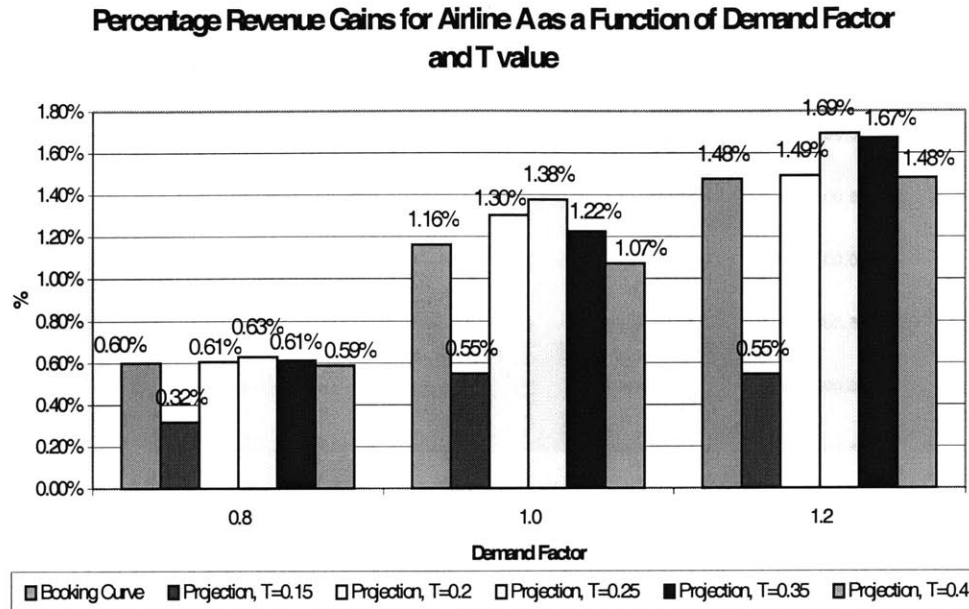
- In the first cases, we will be varying the τ value when airline A is accounting for sell-up and using Projection detruncation while airline B is doing neither.
- In the second set of cases, we will be modifying the τ value when both airlines are accounting for sell-up but only airline A uses Projection detruncation.

4.2.2.b Only Airline A Accounts for Sell-up

In this first section, we will show that the revenue gains actually do improve when we increase the τ value and hence reduce the aggressiveness of Projection detruncation. Moreover, results show that we can increase the revenue gains beyond those obtained with Booking Curve detruncation.

Indeed, as we change the τ value from 0.15 to 0.4, we see that the revenue gains increase for airline A. Moreover, the revenues increase beyond those obtained with Booking Curve detruncation. Figure 19 shows the distribution of revenues as a function of the demand factor and the assumed sell-up rate. In particular, we will now focus on differential input sell-up rates and not so much on constant input sell-up rates. Indeed, we have established that differential input sell-up rates bring the most improvement in revenues.

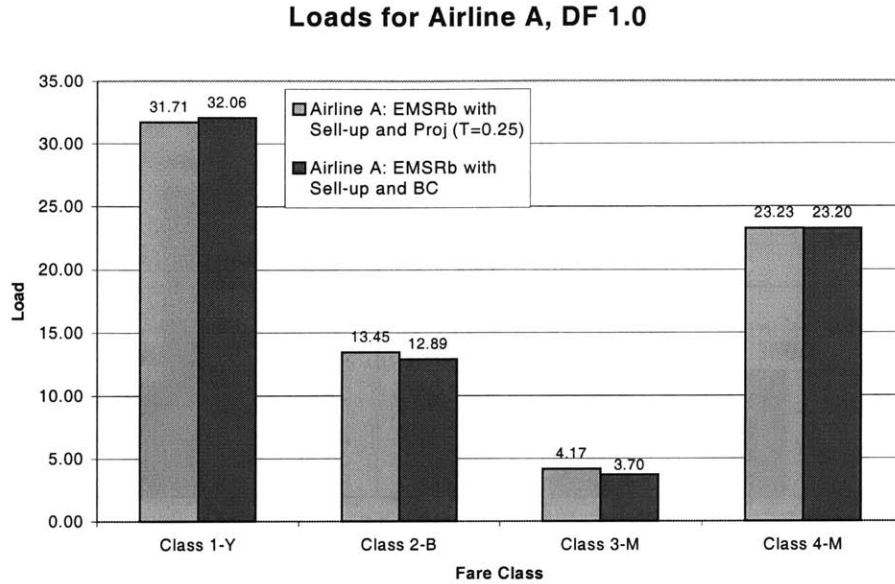
Figure 19: Percentage Revenue Gains for Airline A as a Function of the τ Value and the Demand Factor, Airline A with Differential Sell-up Rates Assumed, Airline B without Sell-up



As the τ value increases from 0.15 to 0.2, 0.25, 0.3, 0.35 and 0.4, we note that there appears to be a peak in revenue gains at a τ value somewhere in between 0.25 and 0.3. In this case, we can then safely assume that we have reached the “optimal” τ value for our simulation. Even though this result by no means gives us an “optimal” τ value applicable in general, it gives us the best in our simulations. In terms of revenue gains, we see that we not only reach a maximum, but that this maximum revenue gain is above the revenue gain achieved with Booking Curve detruncation, at all three tested demand factors. At demand factor 1.0, we see on the Figure 19 that the gains increase by as much as 0.2% (at $\tau=0.25$).

In terms of loads, we show on Figure 20 what happens. The best τ value leads to higher loads in B and M class, slightly higher loads in Q class, while loads are a little lower in Y class than when we were using Booking Curve detruncation. Overall, however, loads are very similar and this explains why the additional revenue gain achieved through Projection detruncation is minimal.

Figure 20: Comparison in Loads for Airline A when it Alternatively Uses Projection Detruncation or not, Combined with Sell-up; Airline B Uses Booking Curve Detruncation without Sell-up, DF 1.0



Overall, the additional revenue gains achieved through Projection detruncation are very small, in the order of 0.2% at the most.

This is in contrast to earlier studies (**Zickus**⁹) that found that switching to Projection detruncation brings clear revenue gains, ranging from 0.15% at demand factor 0.8 to 0.55% at demand factor 1.0 and 0.6% at demand factor 1.2. **Zickus** also found that switching the detruncation method to Projection detruncation had a much greater impact than modifying the Forecaster. Overall, according to **Zickus's** findings, it is beneficial to switch to Projection detruncation. However, in our case, it is not, for the following reasons:

- We are accounting for sell-up and therefore already increasing the protection levels for the higher fare classes,
- Switching to Projection detruncation increases the protection levels, but in a less sensitive way. Indeed, Projection detruncation increases the protection levels in a uniform manner as it detruncates in the same way regardless of the fare class that is concerned. In contrast, sell-up models allow us to apply different sell-up rates to different classes and therefore to better account for small differences in between the classes.

Overall, the combination of these two incremental ways to increase protection levels overlap and may lead to a reduction in revenue gains. The sell-up model allows for more flexibility in terms of the adjustments of the protection levels. Therefore, adding a more aggressive detruncator leads to overprotection.

4.2.2.c Both Airlines Account for Sell-up

Finally, in this last section of this first chapter, we will be discussing the second case: What happens when both airlines account for sell-up and only airline A uses Projection detruncation.

As in the previous paragraph, we get an optimal value for τ , as far as revenue gains are concerned, for airline A. This optimal τ value turns out to be the same as previously, that is, in the vicinity of 0.25-0.3. We were uncertain whether this would be the case or not, as both airlines now better model consumer behavior in terms of accounting for sell-up. Therefore, we would have expected the ensuing protections to be closer to the “reality” of the simulator and hence the τ value to be a little greater than in the previous case.

Moreover, the revenue gains increase again by about 0.2% at demand factor 1.0, as shown in Table 24. This increase is in the same order of magnitude as in the case when only airline A accounts for sell-up. Once again, the additional revenue gain over the case when both airlines account for sell-up is minimal.

Table 24: Percentage Revenue Gains for Airline A when both Airlines are Accounting for Sell-up with Differential Sell-up Rates Assumed, and only Airline A is Using Projection Detruncation

Differential Sell-up Rates At All Demand Factors		
DF	Detruncation	Percentage Revenue Gain
0.8	Booking Curve	0.41%
	Projection (T=0.25)	0.46%
1.0	Booking Curve	0.72%
	Projection (T=0.25)	0.91%
1.2	Booking Curve	1.09%
	Projection (T=0.25)	1.20%

4.2.3 Conclusion

In conclusion to this second section, we note that there is a gain to be achieved through switching to Projection detruncation. This gain was obtained when combined to sell-up gains, at a τ value between 0.25 and 0.3. Moreover, this “optimal” τ value is “optimal” both when only one or both airlines actually account for sell-up.

The revenue gains, however, are in the vicinity of 0.1% to 0.2% over the combination of Booking Curve detruncation and sell-up. That is to say, they are minimal. These revenue gains correspond to an adjustment of the loads, as shown in Figure 20. Loads in B, M and Q class increase a little while loads in Y class decrease slightly, thus generating very small revenue gain over EMSRb with sell-up and Booking Curve detruncation.

4.3 Adjusted Booking Curve

In this section, we now study the impact of our third detruncation method, Adjusted Booking Curve detruncation. The reader will remember that this Adjusted Booking Curve detruncation is nothing more than usual Booking Curve detruncation scaled by a factor between 0 and 1 to account for the fact that the usual Booking Curve algorithm tends to under-estimate the unconstrained demand. Therefore, when we implement this scaling, we effectively detruncate more, thereby increasing the protections in the higher fare classes.

Once again, we will follow the pattern we have been following in the previous sections and present the results when only one airline accounts for sell-up and then when both airlines account for sell-up. In the following sections, airline A is the only airline using this modified detruncation technique.

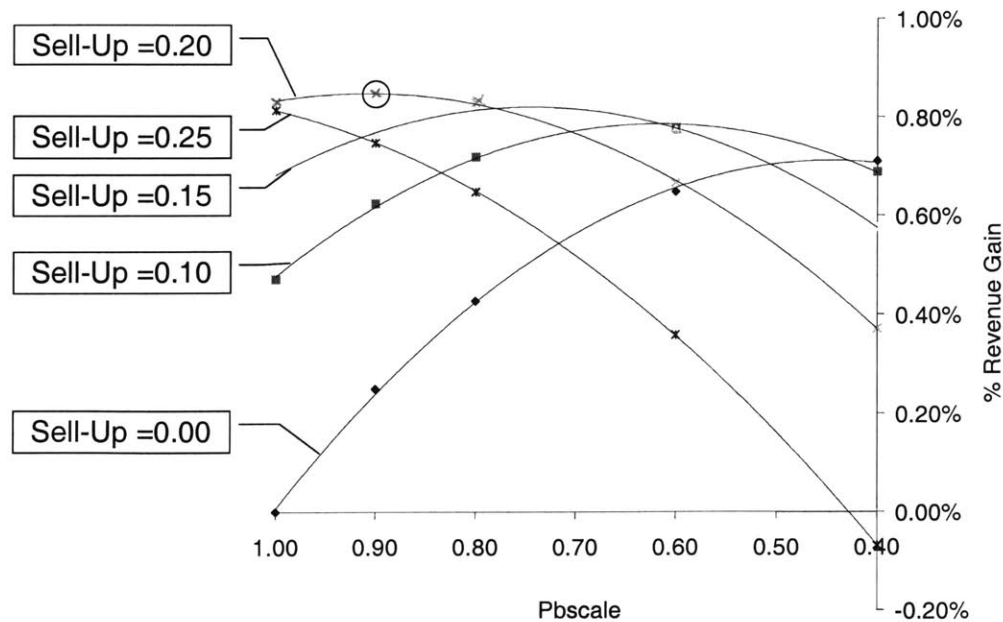
4.3.1 *Only Airline A Accounts for Sell-up*

In this section, we describe the results of our simulations when we use Adjusted Booking Curve detruncation. In particular, we vary the value of the scaling parameter to find the optimal value as a function of the input sell-up rate for airline A. We will see that, in this case, this optimal scaling parameter varies quite a bit as a function of the assumed sell-up rate. However, the highest revenue gains will still be achieved at the same input sell-up rates as earlier and at a scaling factor of 0.9 or 1.0, that is, with very little scaling or no scaling at all.

4.3.1.a *Constant Sell-up Rates*

Figure 21 shows how revenues vary as a function of input sell-up rate and scaling parameter, at demand factor 1.0. It should be noted here that the same trends are observed at demand factors 0.8 and 1.2.

Figure 21: Percentage Revenue Gains as a Function of Assumed Sell-up Rate and Scaling Factor, DF 1.0



We clearly see on this graph that the maximum of all curves, which indicates the maximum revenues, our goal, is located on the 20 percent constant input sell-up rate curve, at a scaling factor of 0.9. Clearly, the revenue gains vary tremendously as a function of the sell-up rates, on the one hand, as shown by the vertical variation at a fixed scaling factor value. This result had already been established earlier with a scaling factor of 1.0 (Booking Curve detruncation). On the other hand, the horizontal scale shows the variation in revenues as a function of the scaling factor. Again, we see on this figure that the revenues, at a given input sell-up rate, vary dramatically as a function of the scaling factor. For example, at a constant input sell-up rate of 20 percent, we reach a revenue gain of 0.85% at a scaling factor of 0.9. However, if we move to a Pbscale value of 0.6, revenue gains decrease to 0.67%. Moreover, it is also interesting to point out that, while it is clearly sub-optimal at a constant input sell-up rate of 20 percent per fare class, this 0.6 Pbscale value is best for a sell-up rate of 10 percent per fare class.

Table 25: Percentage Revenue Gains as a Function of Demand Factor and Pbscale Factor, 20% Sell-up Rate per Fare Class

DF	Pbscale	Revenues		Percentage Gains	
		Airline A	Airline B	Airline A	Airline B
0.8	0.9	\$190,562.00	\$189,286.00	0.39%	-0.22%
	1.0	\$190,559.00	\$189,302.00	0.39%	-0.21%
1.0	0.9	\$228,888.00	\$225,626.00	0.85%	-0.58%
	1.0	\$228,846.00	\$225,731.00	0.83%	-0.54%
1.2	0.9	\$262,206.00	\$258,350.00	0.94%	-0.65%
	1.0	\$262,205.00	\$258,423.00	0.94%	-0.62%

In terms of revenue gains, we note from Table 25 that the revenues are indeed a little higher when we adjust the Booking Curve detruncation method by a factor of 0.9. However, it is also very clear that the difference in revenues is very small and cannot therefore be truly considered as significantly different.

Overall, this first analysis of gains achieved through Adjusted Booking Curve detruncation leads us to the conclusion that, first of all, our “best” constant input sell-up rate of 20 percent per fare class is still the “best” rate even when using Adjusted Booking Curve detruncation. Moreover, the additional gains that can be obtained through this other detruncation method are, at best, minimal.

4.3.1.b Differential Sell-up Rates

In this case, we use our previously defined differential assumed sell-up rates of 45%, 30% and 20% sell-up rates per fare class. The results show that now, clearly, the highest revenue gains are achieved at a scaling factor of 1.0, that is, when no scaling is done. All the results are shown in Table 26.

Table 26: Percentage Revenue Gains as a Function of Demand Factor and Pbscale Parameter at Various Assumed Sell-up Rates

Input Sell-Up DF=0.80 Pbscale	0.00				Differential Sell-Up			
	Airl. A	Airl. B	%wrtBC	%wrtBC	Airl. A	Airl. B	%wrtBC	%wrtBC
0.40	190,329	189,386	0.27%	-0.22%	189,965	189,781	0.08%	-0.02%
0.60	190,292	189,432	0.25%	-0.20%	190,477	189,647	0.35%	-0.09%
0.80	190,116	189,538	0.16%	-0.14%	190,784	189,591	0.51%	-0.12%
0.90	189,970	189,624	0.08%	-0.10%	190,876	189,580	0.56%	-0.12%
No Adj 1.00	189,813	189,703	0.00%	-0.06%	190,948	189,575	0.60%	-0.13%

Input Sell-Up DF=1.00 Pbscale	0.00				Differential Sell-Up			
	Airl. A	Airl. B	%wrtBC	%wrtBC	Airl. A	Airl. B	%wrtBC	%wrtBC
0.40	228,565	225,724	0.71%	-0.54%	227,238	226,706	0.13%	-0.11%
0.60	228,429	225,956	0.65%	-0.44%	228,488	226,479	0.68%	-0.21%
0.80	227,924	226,345	0.43%	-0.27%	229,251	226,428	1.01%	-0.23%
0.90	227,520	226,590	0.25%	-0.16%	229,464	226,465	1.11%	-0.21%
No Adj 1.00	226,954	226,952	0.00%	0.00%	229,593	226,540	1.16%	-0.18%

Input Sell-Up DF=1.20 Pbscale	0.00				Differential Sell-Up			
	Airl. A	Airl. B	%wrtBC	%wrtBC	Airl. A	Airl. B	%wrtBC	%wrtBC
0.40	261,743	258,502	0.76%	-0.59%	259,068	261,146	-0.27%	0.43%
0.60	261,788	258,621	0.78%	-0.54%	261,007	260,531	0.48%	0.19%
0.80	261,248	259,085	0.57%	-0.36%	262,567	260,037	1.08%	0.00%
0.90	260,641	259,491	0.34%	-0.21%	263,135	259,911	1.30%	-0.05%
No Adj 1.00	259,762	260,029	0.00%	0.00%	263,596	259,825	1.48%	-0.08%

Table 26 clearly shows that at all three demand factors, scaling the Booking Curve does not lead to any improvement in the revenues achieved by accounting for sell-up. However, as seen in this same table, when no sell-up is assumed, the “best” scaling factor is close to 0.4-0.5. This confirms that more aggressive detruncation does improve the revenues, when no sell-up is assumed. Overall, we then confirm our hypothesis that detruncation and accounting for sell-up have the same effect:

Increase the seat protection and better match passenger behavior to increase the revenues of the airline.

4.3.2 Both Airlines Account for Sell-up

In this section, we now study what happens to the revenues for airline A when both airlines account for sell-up. In this set of simulations, we set both airlines to account for sell-up while only airline A uses Adjusted Booking Curve detruncation. We see that the revenues do not improve when we increase the detruncation by adjusting the Booking Curve by a scaling factor strictly less than 1.

Table 27 summarizes the results obtained through simulation.

Table 27: Percentage Revenue Gains when Both Airlines Account for Sell-up

		Differential Sell-up Rate vs Differential Sell-up Rate	Revenues		Percentage Gain over Base	
DF=0.8	Pbscale		Airl. A	Airl. B	Airl. A	Airl. B
	0.40		\$189,423.00	\$189,293.00	-0.21%	-0.27%
	0.60		\$189,977.00	\$189,831.00	0.09%	0.01%
	0.80		\$190,332.00	\$190,215.00	0.27%	0.21%
	0.90		\$190,489.00	\$190,349.00	0.36%	0.28%
No Adj	1.00		\$190,598.00	\$190,476.00	0.41%	0.35%
DF=1.0	Pbscale		Airl. A	Airl. B	%wrt BC	%wrt BC
	0.40		\$225,728.00	\$225,756.00	-0.54%	-0.53%
	0.60		\$226,969.00	\$226,993.00	0.01%	0.02%
	0.80		\$227,895.00	\$227,903.00	0.41%	0.42%
	0.90		\$228,273.00	\$228,269.00	0.58%	0.58%
No Adj	1.00		\$228,592.00	\$228,613.00	0.72%	0.73%
DF=1.2	Pbscale		Airl. A	Airl. B	%wrt BC	%wrt BC
	0.40		\$258,894.00	\$259,267.00	-0.33%	-0.29%
	0.60		\$260,288.00	\$260,652.00	0.20%	0.24%
	0.80		\$261,495.00	\$261,885.00	0.67%	0.71%
	0.90		\$262,065.00	\$262,451.00	0.89%	0.93%
No Adj	1.00		\$262,584.00	\$262,942.00	1.09%	1.12%

All the scaling factors tested here do not bring any improvement to the revenue gains from EMSRb with sell-up under regular Booking Curve detruncation. The reason for this can partly be explained by our previous results. Indeed, we know that when we apply a scaling factor less than 1, we increase the detruncation. This phenomenon can be compared to reducing the τ value with Projection detruncation. Our analysis of Projection detruncation has led us to believe that there is very little improvement to be gained by increasing the detruncation. Adjusted Booking Curve detruncation is another way to switch to more aggressive detruncation. The goal of the method remains the same: To increase the aggressiveness of the detruncation. It is therefore quite logical that we should get the same results as with Projection detruncation, even though the method itself is different.

4.3.3 Conclusion

In this section, we have seen that, regardless of the competitive situation in terms of accounting for sell-up, there is very little, if anything, to be gained from switching detruncation methods from BC detruncation to Adjusted Booking Curve detruncation. Unlike Projection detruncation where we achieved revenue gains of about 0.1% above our initial gains obtained by accounting for sell-up, we have no consistent evidence that there is any gain to be achieved with Adjusted Booking Curve detruncation. In particular, our analysis shows that the best results are obtained with Booking Curve detruncation (which is the extreme case Adjusted Booking Curve, when the scaling parameter is set to 1).

4.4 Summary

In this chapter, we studied the possible revenue gains that can be achieved by accounting for sell-up in a Fare Class Yield Management environment. Our simulations show that we can expect revenue gains as high as 1.5% when only one airline is accounting for sell-up and as high as 1.1% when both airlines account for sell-up, at demand factor 1.2. Moreover, it appears that the potential revenue gains depend on the demand factor, but, whatever the demand factor, the gains are positive. In addition, we also noted that airline A maximizes its revenues when it uses a differential input sell-up rate in the EMSRb heuristic. At the same time, airline B is also better off when its competitor uses a differential input sell-up as opposed to constant input sell-up rates. As logic suggests, when both airlines account for sell-up, the revenue gains decrease for airline A, as both airlines are now better at protecting their inventory.

Finally, we examined Projection detruncation and Adjusted Booking Curve detruncation in an attempt to further improve our gains, if possible. Our conclusion is that accounting for sell-up and modifying the detruncation method amount more or less to the same result in terms of booking limits and protections. However, accounting for sell-up provides more flexibility in terms of where to protect more or less. This is shown by the differential sell-up rates that are designed to match passenger willingness-to-pay.

Through simulations, we found out that the “best” τ value was in the vicinity of 0.25-0.3, and allowed revenue increases of at most 0.2% over the gains achieved by accounting for sell-up. However, as far as Adjusted Booking curve detruncation is concerned, we noted that the “best” scaling factor was 1, that is, when no adjustment was made. Overall, we also pointed out that the revenue gains obtained by switching detruncation methods are minimal.

Chapter 5: Sell-up in a Virtual Bucket Environment

In this second chapter of Part II, we will now be dealing with a virtual bucket environment in our Revenue Management system. In particular, airline A will always be using Greedy Virtual Nesting (GVN, c.f. Chapter 1.2.2.a) as its Revenue Management algorithm. In addition, it should be reminded here that our base case has not changed and remains EMSRb vs. EMSRb as described at the beginning of this second part of the thesis.

This chapter will be devoted to understanding and explaining what happens when one of the airlines, or both, accounts for sell-up in a more sophisticated Revenue Management environment, namely GVN. It has been established in previous research, including Lee⁸, that switching to a virtual bucket environment brings revenue gains ranging from 0.7% at demand factor 0.8 to 1.6% at demand factor 1.0 and 1.4% at demand factor 1.2. We note here that, as we explained in Part I, Chapter 1, on Revenue Management methods, GVN does not perform quite as well at high demand factors, as it favors connecting passengers over local passengers. The reader will recall that this means that two local passengers, who individually pay a lower fare, may be displaced in favor of a connecting passenger who pays a higher fare. However, the sum of these two lower local fares may be higher than the connecting fare, hence reducing the overall revenue gain for the airline. This problem becomes more critical as demand increases.

Our goal in this chapter, independently of this limitation of GVN, is to evaluate the possible revenue gains that can be achieved if the airline using GVN accounts for sell-up. We established in the previous chapter that, in a Fare Class Yield Management environment, accounting for sell-up is clearly beneficial to the airline. The revenue gains range from 0.6% to 1.5% when only one airline is accounting for sell-up and both are using Booking Curve detruncation. The question we raise in this chapter is whether we can improve the revenue gains when the airline both accounts for sell-up and uses a Revenue Management algorithm designed to perform basic O-D control.

The structure of this chapter will be very similar to that of Chapter 1 in that we will first focus on what happens when we use Booking Curve detruncation, describe the results and potential revenue gains, and then switch to Projection detruncation and see what happens in this case.

It is important that the reader remember here what we mean by input or assumed sell-up rates: These are the sell-up rates that we input in the **Belobaba-Weatherford**²² heuristic. These input sell-up rates are estimated by us to match as best as possible the propensity of a passenger to sell up to a higher fare class.

5.1 Booking Curve Detruncation

In this section, we will be focusing on the potential revenue gains achievable when one airline or both airlines account for sell-up and use Booking Curve detruncation in conjunction with GVN. As our results will show, accounting for sell-up clearly leads to revenue gains over our EMSRb vs. EMSRb base case. In the following paragraphs, we will first focus on the effect of accounting for sell-up when airline B uses Fare Class Yield Management (FCYM) and we will then look at the revenue gains that can be achieved when both airlines use GVN as their Revenue Management system. Finally, we will study how gains vary with the competitive situation. However, we will start by defining the notion of input sell-up rates and in particular differential input sell-up rates in a virtual bucket environment.

5.1.1 GVN vs. EMSRb

In this first section we will be focusing on the impact of accounting for the possibility of sell-up when airline A uses GVN and is competing against an airline that only uses Fare Class Yield Management (FCYM), typically EMSRb, with or without accounting for the possibility of sell-up. We will see that whether or not airline B accounts for sell-up, airline A nonetheless benefits from accounting for sell-up, compared to our base case scenario (EMSRb vs. EMSRb).

5.1.1.a Only Airline A Accounts for Sell-up

As described above, in this section, we focus on the revenue gains achieved by airline A when it is the only airline accounting for sell-up. We always compare our results to the base case. The simulations show that the revenue gains are now very high. They reach as much as 3.7% over base EMSRb without sell-up assumed at demand factor 1.2 and differential sell-up rates assumed.

We divide this section into two sub-sections dealing respectively with constant input sell-up rates and then with differential input sell-up rates. We will see in the following paragraphs that, as we established with EMSRb Revenue Management, differential input sell-up rates, when well chosen, lead to higher revenue gains than constant input sell-up rates.

Constant Input Sell-up Rates

Given that GVN is different from FCYM in that it favors long-haul connecting passengers over short-haul local passengers, and since it spreads demand into more buckets, we expect the “best” constant input sell-up rates, with GVN, to be lower than with FCYM: Indeed, the algorithm will tend to “blow-up” earlier with GVN than with FCYM. We therefore first tested different input sell-up rates to confirm or reject this hypothesis. Table 28 shows the “best” sell-up rates assumed at all three demand factors. We note here that constant sell-up rates, in a virtual bucket environment, are simply the probability that a passenger will be willing to sell-up from one bucket to the next.

Table 28: Revenue Gains as a Function of Demand Factor and Input Sell-up Rate

		Absolute Revenue Gain		Percentage Gain over EMSRb Base	
DF	Input Sell-up Rate	Airline A	Airline B	Airline A	Airline B
0.8	0%	\$1,302.00	-\$1,093.00	0.69%	-0.58%
	10%	\$1,874.00	-\$1,415.00	0.99%	-0.75%
	20%	\$1,726.00	-\$1,931.00	0.91%	-1.02%
	30%	-\$4,140.00	-\$1,749.00	-2.18%	-0.92%
1.0	0%	\$3,575.00	-\$2,148.00	1.58%	-0.95%
	10%	\$4,669.00	-\$2,801.00	2.06%	-1.23%
	20%	\$5,173.00	-\$3,403.00	2.28%	-1.50%
	30%	\$2,383.00	-\$3,790.00	1.05%	-1.67%
1.2	0%	\$3,503.00	-\$1,654.00	1.35%	-0.64%
	10%	\$5,946.00	-\$2,833.00	2.29%	-1.09%
	20%	\$8,139.00	-\$4,049.00	3.13%	-1.56%
	30%	\$5,439.00	-\$4,998.00	2.09%	-1.92%

We observe in Table 28 that airline A achieves revenue gains at all three demand factors when accounting for sell-up. In particular, we observe a peak in revenue gains at 10 percent input sell-up rates and demand factor 0.8, with 1% gains, while at demand factors 1.0 and 1.2, the peak is reached at 20 percent input sell-up rates and is respectively 2.3% and 3.1%. A possible explanation for the difference in “best” input sell-up rates as a function of demand factor may be in the virtual buckets. As we introduce these buckets and input a sell-up rate into the algorithm, at low demand factors, the lower buckets are going to close down earlier than without sell-up assumed, and this will have a higher impact on revenues as it will keep the higher buckets open and therefore allow for most of the long haul passengers to find available seats, especially discount seats as they will book first. Therefore, the high yield, late booking, local passengers will find less availability. Overall, the impact of denying access to these passengers will then be higher on revenues, which is what we observe here. This is the same problem, but translated to demand factor 0.8, as the usually observed disadvantage of GVN, which favors connecting passengers over local passengers and therefore has a greater tendency to have lesser revenue gains at higher demand factors.

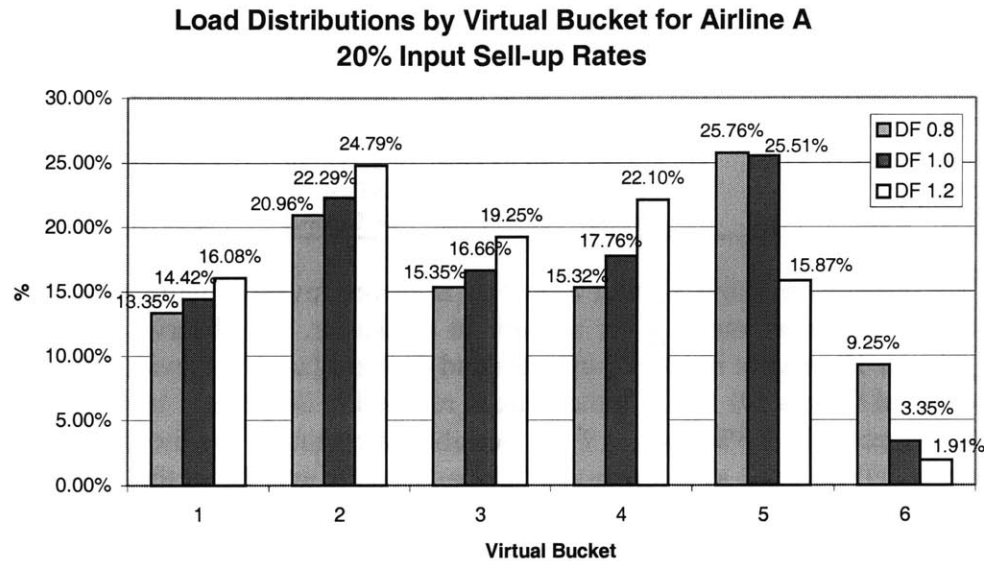
Similarly, we also note that at 30 percent input sell-up rates, we observe lower gains at all three demand factors. At demand factor 0.8, the algorithm is hugely overprotecting, while at demand factors 1.0 and 1.2 we have lower revenues but still positive gains. However, once again, at these demand factors, we are also overprotecting.

In terms of loads, we now focus on virtual bucket loads. Indeed, airline A now uses virtual bucket fare mapping, whereas airline B still uses EMSRb fare class mapping. For convenience purposes, we will map the bookings for both airlines according to virtual buckets.

We first look at the distribution of passengers between virtual buckets for airline A at a constant input sell-up rate of 20% per virtual bucket. We note on Figure 22 that

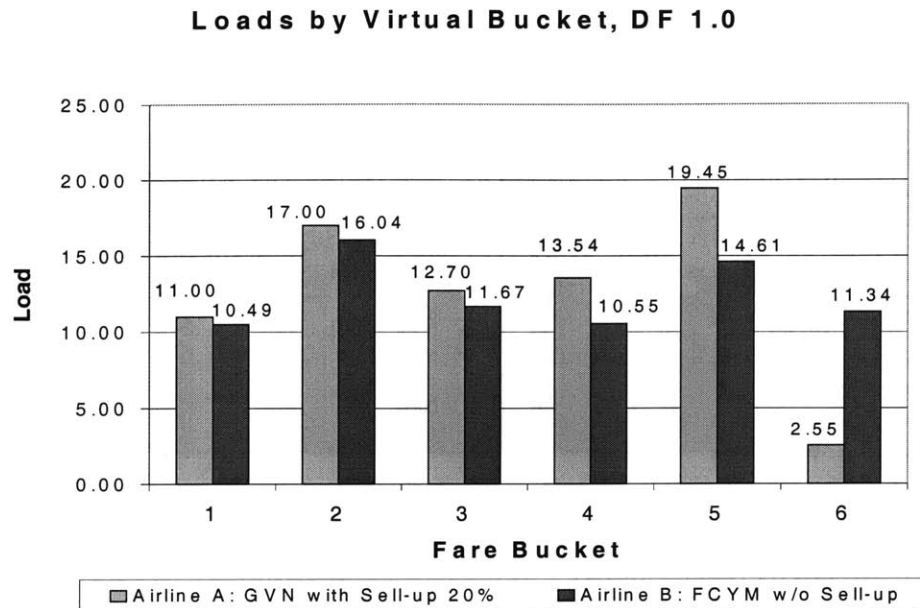
the load distribution increases with demand in the first four buckets whereas the loads decrease in the last two buckets as demand increases. This points out the fact that at high demand factors, GVN protects more for connecting passengers. In addition, absolute loads increase with demand (not shown on Figure 22).

Figure 22: Load Distributions by Virtual Bucket for Airline A at a Constant Input sell-up Rate of 20% per Virtual Bucket



As far as comparing the loads between airlines, Figure 23 shows the loads by virtual bucket for both airlines at demand factor 1.0 and a constant input sell-up rate of 20 percent per virtual bucket. It is clear on Figure 23 that the loads are substantially higher for airline A in the first five buckets and lower in the last bucket. This shows that when airline A is accounting for sell-up, it rejects a lot more low fare passengers and spills them to the competing airline, overall increasing its revenues by getting more high fare passengers. Conversely, airline B is less efficient in protecting its seats and fills up with low fare passengers.

Figure 23: Loads by Virtual Bucket at Demand Factor 1.0 and 20% Input sell-up Rate



Finally, in Figure 24, we take a closer look at what happens when revenues start decreasing. Figure 24 shows the load distributions by virtual buckets for airline A at demand factor 0.8 as a function of the input sell-up rate. We see on Figure 24 that the passenger distribution according to virtual buckets is such that the percentage of passengers in the first five buckets increases with the input sell-up rate. However, the amount of passengers in the last bucket decreases much faster and leads to huge revenue losses. Moreover, this chart does not show the absolute value of the loads but the distribution of the loads. Hence we do not see on Figure 24 that the average load factor decreases with the input sell-up rate, and that the differences actually increase in terms of absolute loads.

Figure 24: Passenger Distribution by Virtual Bucket as a function of the Input sell-up Rate, DF 0.8

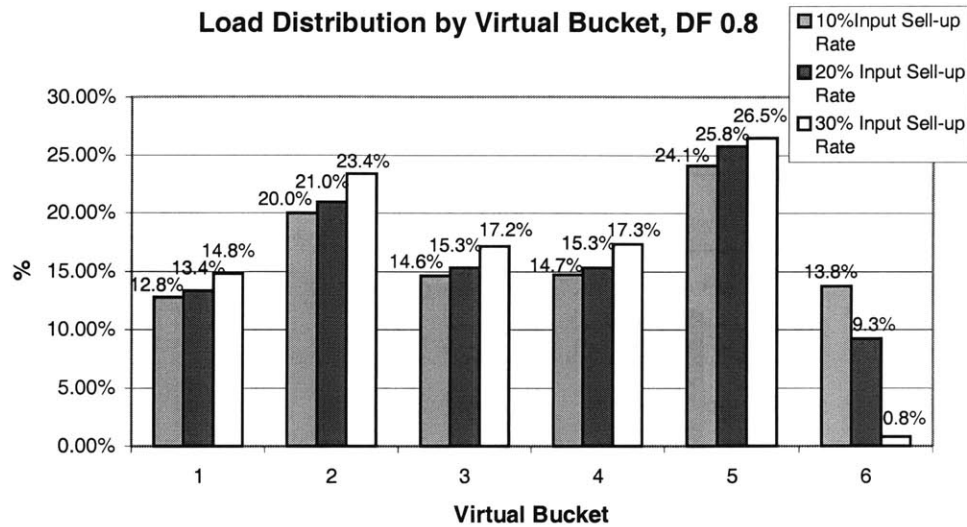
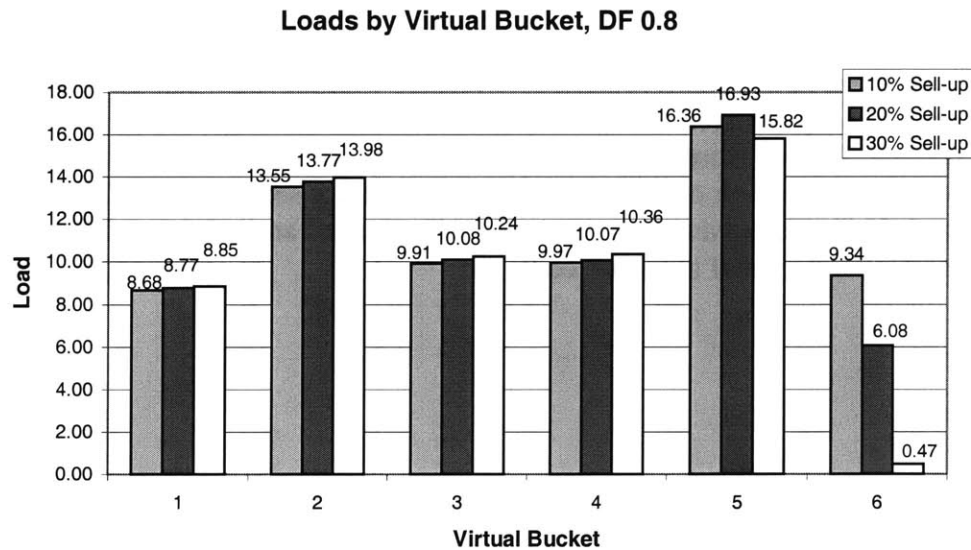


Figure 25: Virtual Bucket Loads at Demand Factor 0.8, as a Function of the Input sell-up Rate



Finally, Figure 25 shows the absolute loads and confirms the fact that the general trend in loads in the first four buckets is that loads are higher as the input sell-up rate increases. However, in the last two buckets, loads decrease at 30% input sell-up rate. In particular, in the lowest bucket, the load is extremely low, thus explaining the overall loss in revenues. As far as the 20% input sell-up rate case is concerned, what happens is that the losses incurred as a result of the lower loads in the last two

buckets outweigh the gains in the higher four buckets, thus leading to an overall reduction in revenues.

In summary, all these results show that the combination of sell-up models with GVN, when opposed to an airline that uses EMSRb (or Fare Class Yield Management, a.k.a. FCYM) without accounting for sell-up, always brings revenue gains ranging from 1% to about 3.1% depending on the demand factor, at 20% input sell-up rate at demand factors 1.0 and 1.2, and at 10% input sell-up rate at demand factor 0.8.

Differential Input sell-up Rates

In this paragraph, we now focus on differential input sell-up rates, as we once again expect them to lead to higher revenue gains, as **Bohutinsky**²⁰ suggested. We will see that the results confirm this hypothesis. However, it is more difficult to justify our choice of input sell-up rates, and we will spend the first paragraph explaining this choice.

Choosing a Differential Sell-up Rate

Bohutinsky²⁰ suggests that it is reasonable to believe that business and leisure passengers do not have the same willingness-to-pay and have different likelihood to sell up or buy a higher fare ticket when their first choice is not available. However, in the case of virtual buckets, fares are mapped according to their dollar value. Therefore, a given virtual bucket may contain a short-haul Y along with fare with a long-haul M fare, while another bucket may only have B class long-haul fares, etc. Overall, it now becomes very difficult to distinguish between passenger types and explain willingness to sell up by the fare bucket. In the extreme buckets, one can argue that we have mostly high fare passengers and low fare passengers, depending on the extremity of the bucket. It would therefore be easier to justify a choice of differential input sell-up rates in these extreme buckets.

In accordance with **Bohutinsky**'s results²⁰, we picked the same type of sell-up rates as earlier, and decided to run tests on three particular combinations:

- 45%-35%-30%-20% and 10% per virtual bucket, from top down,
- 40%-35%-30%-20% and 10% per virtual bucket, and,
- 35%-30%-25%-15% and 5% per virtual bucket.

The results are shown on Table 29.

Table 29: Revenue Gains as a Function of Differential Input Sell-up Rate, DF 1.0

DF	Input Sell-up Rate	Absolute Revenue Gain		Percentage Gain over EMSRb Base	
		Airline A	Airline B	Airline A	Airline B
1.0	40-35-30-20-10%	\$6,427.00	-\$3,410.00	2.83%	-1.50%
	45-35-30-20-10%	-\$4,862.00	\$1,792.00	-2.14%	0.79%
	35-30-25-15-5%	\$5,847.00	-\$3,112.00	2.58%	-1.37%

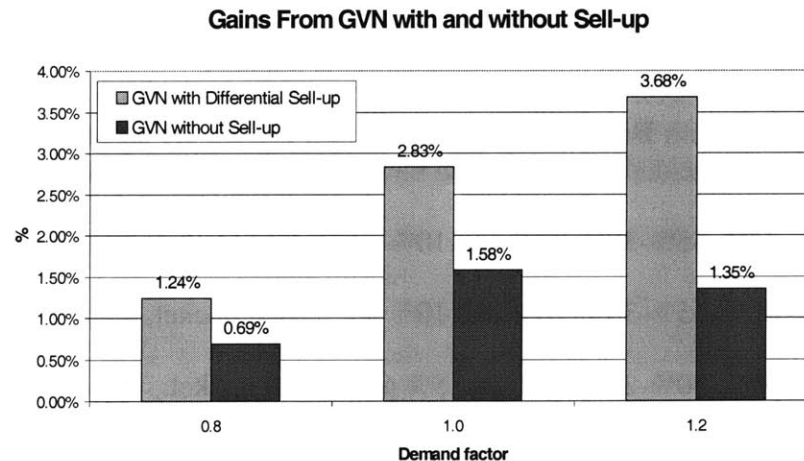
We clearly see in Table 29 that the first input sell-up rate tested brings the highest revenue gains. We therefore decided to keep this as the “differential input sell-up rate” for the remainder of the thesis, at least in terms of assumed inputs to GVN.

Results

Revenues

As suggested in Table 29, we get very high revenue gains when we account for sell-up and input a differential sell-up rate in the EMSRb Sell-up Heuristic as part of the GVN Revenue Management algorithm. Depending on demand factor, revenue gains range from 1.2% to 3.6%. In terms of gains over the initial GVN algorithm, Figure 26 shows the differences as a function of demand factor. We note here that the difference increases with the demand factor. Moreover, the lesser performance of GVN at high demand factors that can be seen on Figure 26, when airline A is not accounting for sell-up, seems to have disappeared when we account for sell-up.

Figure 26: Comparison between Gains from GVN with and without Sell-up when the Competitor Does Not Account for Sell-up



Overall, the revenue gains are substantial and almost double the gains achieved by using GVN without sell-up. Another interesting point is that, similar to the EMSRb case, airline B, which is using the basic Fare Class Yield Management method, happens to be losing less money when its competitor switches to differential input

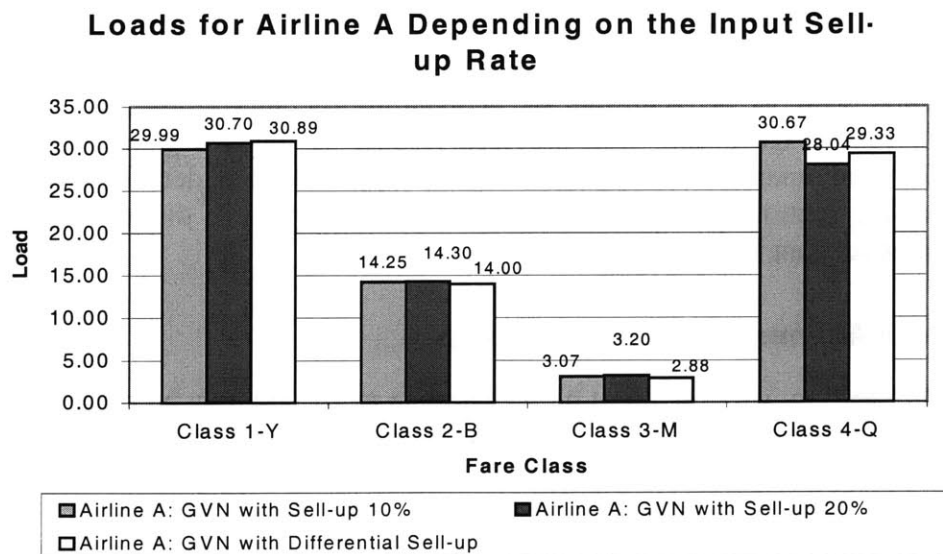
sell-up rates. The reason for this is that airline A is now trying to match its protections to passenger behavior more closely. The result is that airline A achieves lower load factors, but gets more Y and B class passengers and spills more M and Q class passengers. Overall, airline B benefits from this in that its loads are higher than when airline A uses a constant input sell-up rate of 20 percent per virtual bucket. We will get more into the details of this in the following paragraph.

Loads

In this paragraph, we briefly go into the details of the loads as a function of the input sell-up rate. Consistent with all our previous results, we will see that when airline A accounts for sell-up, the loads by fare class increase with the input sell-up rate in Y class. However, they decrease in Q class. This is consistent with the fact that at high input sell-up rates, we overprotect. As shown on Figure 27, a differential input sell-up rate brings the highest revenues, and this is due to the fact that it reaches the highest loads in Y class, while maintaining reasonable loads in the other three fare classes, and in particular in Q-class. We observed, in earlier results, that the greatest difference in loads between high and low demand factors resides in the fact that the loads in Q class are very low when using high input sell-up rates, as we are highly overprotecting. This difference in loads leads to a substantial difference in revenues.

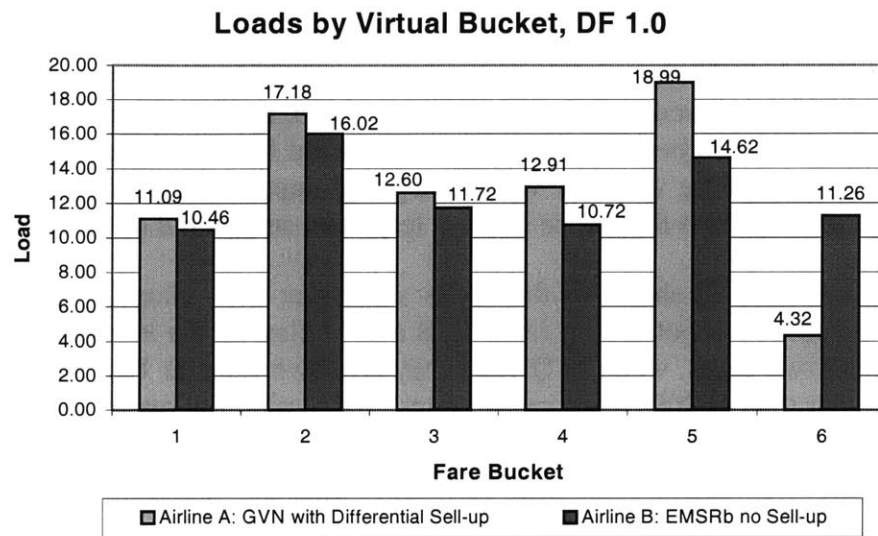
Overall, Figure 27 shows us that loads are higher in Y class for airline A when it uses differential input sell-up rates. In B and M classes, the loads are very similar to the other two cases, while in Q class they are lower than at 10% input sell-up rates, but higher than at 20% input sell-up rates. We remind the reader here that 20% sell-up rates assumed led to the best results at constant input sell-up rates and demand factor 1.0.

Figure 27: Loads for Airline A as a Function of the Input Sell-up Rate, DF 1.0



In terms of comparing the loads of airline A to those of airline B, Figure 28 shows the loads by virtual buckets. As before, we observe that the loads for airline A are higher in the first five buckets but lower in the last bucket. The data also reveals that the average leg load factor for airline A is higher than for airline B. This is a major difference from the previous case. Indeed, when both airlines were using EMSRb, the average leg load factor for airline B was higher than for airline A at differential input sell-up rates. However, as was the case for EMSRb with sell-up, the average leg load factor for airline A has decreased from the “no sell-up assumed” case, as airline A is now protecting more for higher fare passengers.

Figure 28: Loads by Virtual Bucket, Airline A with Differential Sell-up Assumed, Airline B w/o Sell-up, DF 1.0



5.1.1.b Conclusion

This first look at GVN accounting for sell-up shows us that there is a substantial revenue gain to be achieved by accounting for sell-up in this virtual bucket environment. Consistent with previous results, the highest gains are reached at a differential input sell-up rate, and can go as high as 3.7% at demand factor 1.2. In the following section we will be focusing on what happens to the revenues when both airlines account for sell-up.

5.1.2 Both Airlines Account for Sell-up

In this section, we now focus on the revenue gains that may be achieved when both airlines account for sell-up. This set of simulations opposes airline A using GVN with sell-up to airline B using EMSRb with sell-up. When using constant input sell-up rates, we will be using exactly the same input sell-up rates by fare class and by virtual bucket. When using differential input sell-up rates in the **Belobaba-Weatherford** heuristic²², we will be using what we determined to be the best rates in

the previous paragraphs. That is, for the EMSRb method, we will use 45%-30% and 20% per fare class, and, for GVN, we will be using 40%-35%-30%-20% and 10% per virtual bucket, from top down.

Once more, simulations show that there is revenue to be gained by accounting for sell-up in this specific competitive situation. In particular, we observe that airline A achieves revenue gains as high as 3.3% at a differential input sell-up rate combination, while airline B loses money, but comparatively less than when it was not accounting for sell-up.

In the following paragraphs, we will first present the revenue gains (or losses), and then take a closer look at the loads achieved by the two airlines, in order to explain the observed phenomena.

5.1.2.a Revenues

As stated in the previous paragraph, airline A still achieves revenue gains when its competitor upgrades its Revenue Management algorithms to account for the possibility of sell-up. However, consistent with previous results on EMSRb, and as we would expect, as the competitor upgrades its Revenue Management method to account for sell-up, it increases its share of high yield passengers and, therefore, the revenues of airline A decrease compared to the case when airline B did not account for sell-up.

Table 30 shows the revenue gains for both airlines depending on airline B's accounting for sell-up or not. It appears clearly in this table that the revenue gains drop for airline A as the competitor begins accounting for sell-up. On the other hand, revenue losses also decrease for airline B when it accounts for sell-up. Once again, differential input sell-up rates yield the best results for both airlines in both situations.

Table 30: Percentage Revenue Gains as a Function of Demand Factor, Competitive Situation and Input Sell-up Rates

DF	Sell-up Rate	Percentage Gain over EMSRb Base			
		Airline A GVN SU vs FCYM no SU	Airline A GVN SU vs FCYM with SU	Airline B FCYM w/o SU vs GVN SU	Airline B FCYM with SU vs GVN SU
0.8	20%	0.91%	0.78%	-1.02%	-0.72%
	Differential	1.24%	1.20%	-0.86%	-0.45%
1.0	20%	2.28%	1.88%	-1.50%	-0.84%
	Differential	2.83%	2.53%	-1.50%	-0.67%
1.2	20%	3.13%	2.55%	-1.56%	-0.71%
	Differential	3.68%	3.30%	-1.65%	-0.47%

These results also show that the impact of the change in airline B's Revenue Management method is less at demand factor 0.8 for both airlines, while at demand factors 1.0 and 1.2, the effect on airline A's revenues is a loss of about 0.3% in both cases, at differential input sell-up rates. Airline B, on the other hand benefits more

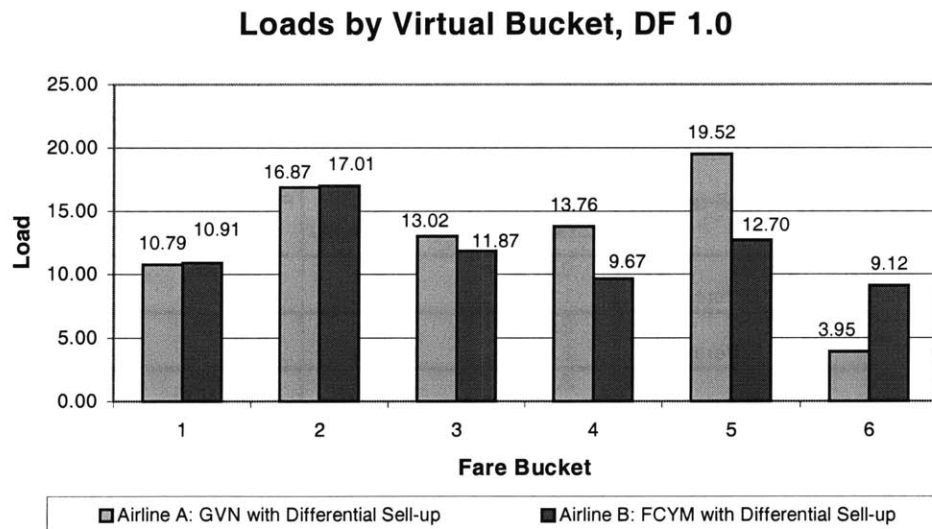
and more as the demand factor increases. This can be explained by the fact that, as demand increases, B is able to attract more high yield passengers, and hence reduces its losses (compared to the EMSRb vs. EMSRb base case). Moreover, as we discussed earlier, since airline A is using GVN, it will tend to protect more for connecting passengers who bring higher revenues (on a per passenger basis) but can be lower yield passengers on each leg, compared to local passengers. Therefore, airline B will get more of the local, higher yield traffic, and thus increase its revenues, to overall reduce its losses.

5.1.2.b Loads

Looking at loads allows us to understand the difference in revenues between airline A and airline B on the one hand and the difference in revenue gains depending on the input sell-up rates on the other hand.

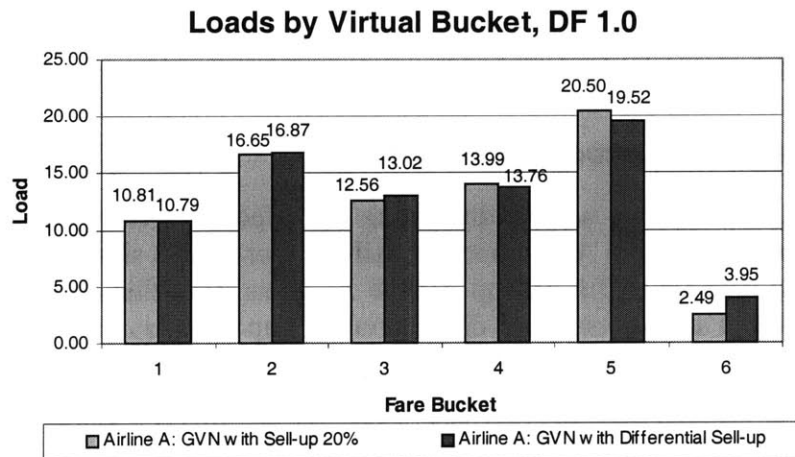
First of all, Figure 29 shows that airline A has substantially higher loads in buckets 3, 4 and 5, while in buckets 1 and 2, airline B has slightly higher loads. In bucket 6, airline B has much higher loads than airline A. In terms of average leg load factor, airline A achieves 78% load factor while airline B is at 71%, at demand factor 1.0. Lower average load factors combined with substantially lower loads in the intermediate virtual buckets leads to the revenue differences we observed in the previous paragraph (5.1.2.a above). Indeed, airline B is slightly better at attracting the very high revenue passengers (in buckets 1 and 2) but the overall loads, and the fact that airline A gets most of the average revenue generating traffic while airline B is overwhelmed by the very low revenue traffic, are responsible for the large revenue difference between the two airlines.

Figure 29: Loads by Virtual Bucket, Demand Factor 1.0, Both Airlines Account for Sell-up



Finally, Figure 30 compares the loads by virtual bucket for airline A, depending on the input sell-up rate, constant or differential, when airline B is accounting for the possibility of sell-up. We observe that the major difference is that airline A achieves higher loads in the first three buckets and the last bucket. The average leg load factor is very similar for both cases: 78% in the differential case and 77% in the constant input sell-up rates case (at demand factor 1.0). Therefore, we can conclude from Figure 30 that the major difference in revenues between these two cases comes from the fact that loads are “better” distributed when we input differential sell-up rates. By “better” distributed, we mean that even though in both cases airline A gets roughly the same amount of passengers, on average, the number of high revenue generating passengers is higher at differential input sell-up rates.

Figure 30: Loads by Virtual Bucket for Airline A, depending on the Assumed Sell-up Rate



5.1.3 Conclusion

This first section allowed us to determine whether there were revenue gains to be obtained from accounting for sell-up in the GVN case, and how much gain could be achieved. It turns out that whether or not airline B is accounting for sell-up in a FCYM case, there is revenue to be gained by accounting for sell-up in the GVN environment. These gains range from 1.2% to 3.7%, depending on the demand factor, with differential input sell-up rates when only airline A accounts for sell-up. When both airlines account for sell-up, these gains drop to 1.2% to 3.3% with differential input sell-up rates. We also looked at loads and explained the relationship between loads and revenue gains or losses for both airlines.

5.2 GVN vs. GVN

In this section, we now focus on the revenue gains achieved by airline A when its competitor also uses GVN with or without sell-up. We will see in the next

paragraphs that airline A still benefits from accounting for sell-up. However, its revenue gains will drop considerably, as we will describe in the following paragraphs. For consistency purposes, we keep comparing the revenues to our base case. However, the reader should also keep in mind that the revenue gains over our base case, when both airlines use GVN but do not account for sell-up, are the following:

- 0.09% at demand factor 0.8
- 0.50% at demand factor 1.0, and
- 0.42% at demand factor 1.2.

This section will, as previously, be divided into two subsections: The first subsection will deal with the case when only airline A accounts for sell-up. We will discuss revenues and loads by virtual buckets. In the second subsection, we will focus on the impact of the competitor's move towards better Revenue Management by accounting for sell-up.

5.2.1 Only Airline A Accounts for Sell-up

In this subsection, we will be discussing the effect of accounting for sell-up in the case when only airline A accounts for sell-up. Our results show that the airline using GVN and accounting for sell-up (airline A) gains from this change. As previously, the best input sell-up rates are differential sell-up rates, as can be seen in Table 31 that summarizes the revenue gains. The differential input sell-up rates that are used in the following paragraphs are the same as previously defined, that is, 40%-35%-30%-20% and 10% per virtual bucket, from top down.

The “best” input sell-up rates, that is, the rates that led to the highest revenue gains were the following:

- 10% for constant input sell-up rates, and,
- 40%-35%-30%-20% and 10% per virtual bucket for differential input sell-up rates.

Above these rates, we see that the revenues start deteriorating and finally turn into losses as airline A's Revenue Management system starts overprotecting for the higher classes.

As shown in Table 31, the revenue gains for airline A range from 0.6% at demand factor 0.8 to 1.6% at demand factor 1.0 and 2.5% at demand factor 1.2 with differential input sell-up rates. Once again, it clearly appears in Table 31 that the benefits of differential input sell-up rates are substantially higher than those of constant input sell-up rates. In particular, at demand factor 1.2, the percentage gains more than double when airline A switches to differential input sell-up rates.

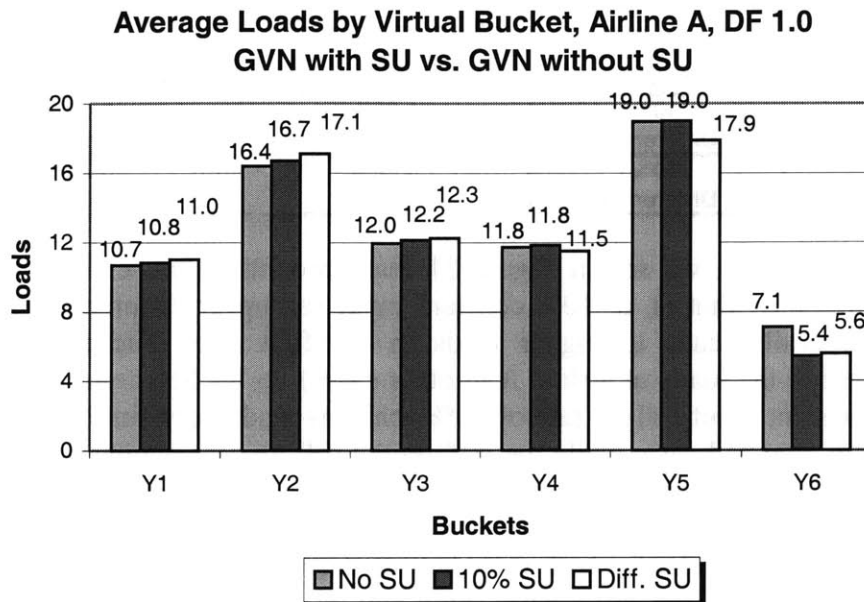
Table 31: Percentage Revenue Gains for Airline A as a Function of the Demand Factor and Input Sell-up Rates, GVN with Sell-up vs. GVN

Demand Factor	Input Sell-up Rates	Percentage Revenue Gain Over Base Case	Percentage Revenue Gains over GVN vs. GVN
0.8	10%	0.35%	0.26%
	Differential	0.56%	0.47%
1	10%	0.95%	0.45%
	Differential	1.58%	1.08%
1.2	10%	1.20%	0.78%
	Differential	2.51%	2.09%

In terms of loads, we see in Figure 31 that accounting for sell-up (in a revenue generating way, that is, at 10% constant input sell-up or differential input sell-up rates) generally leads to higher loads in the first three buckets for airline A (compared to the loads of airline A in other cases). In buckets four and five, we see that a constant input sell-up rate of 10% achieves loads very similar to those of the no sell-up assumed case, while the differential sell-up case achieves lower loads. In the lowest bucket, both input sell-up rates cases have approximately the same loads, substantially lower than those of the “no sell-up” case. Moreover, the average load factors are very similar for both input sell-up cases (75.9% in the 10% input sell-up case and 75.4% in the differential input sell-up case at demand factor 1.0). This allows us to conclude that the revenue difference between the two sell-up cases lies mostly in the distribution of passengers amongst the various fare buckets.

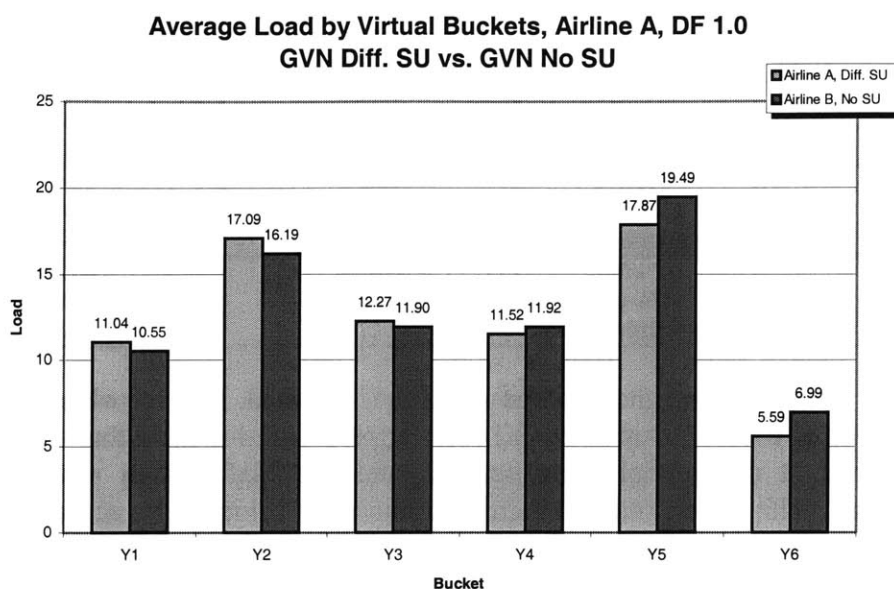
Compared to no sell-up assumed, the average leg load factors decrease in both sell-up cases, consistently with what we experienced with EMSRb and what we would expect. The average leg load factor in the no sell-up assumed case is about 77%.

Figure 31: Load Distribution by Virtual Bucket for Airline A, Depending on the Assumed Sell-up Rate, DF 1.0



Finally, our last analysis of loads concerns the difference in loads between airline A and airline B when airline A accounts for sell-up. Figure 32 shows the loads by virtual bucket. We see on this chart that the loads for airline A are again higher in the first three buckets and lower in the last three buckets. Moreover, the average leg load factor for airline A is about 75%, while airline B has an average leg load factor of about 77% (at demand factor 1.0). Therefore, we can conclude from these two indicators that the revenue difference, to the benefit of airline A, is due to the higher loads in the first three buckets. More low fare passengers are spilled towards airline B, while airline A increases its “high revenue generating” passenger loads.

Figure 32: Loads by Virtual Bucket, GVN with Sell-up vs. GVN w/o Sell-up, DF 1.0, Differential Sell-up Rates Assumed



A similar pattern is observed at 10% input sell-up rates, but in that case, the loads achieved by airline A are a little lower, and those of airline B a little higher, which explains the lower difference in revenues between airlines A and B.

In summary, we have seen in this paragraph that, once again, as both airlines use a virtual bucket Revenue Management scheme, but only one accounts for sell-up, the revenues for the airline accounting for sell-up increase and reach a peak at differential input sell-up rates. These results compare to those described with EMSRb in the previous chapter. However, the gains are much higher. There are two reasons for this: First of all, both airlines are using a basic O-D control Revenue Management algorithm, namely GVN, which is more likely to reach higher revenues. Second, the network layout is such that it favors GVN over EMSRb. We will explain this in more detail at the end of this chapter on GVN.

5.2.2 Both Airlines Account for Sell-up

In the following paragraphs, we now study what happens to the revenues of airline A when both airlines account for the possibility of sell-up in a virtual bucket environment. Once again, we will try various cases, and in particular constant input sell-up rates and differential input sell-up rates.

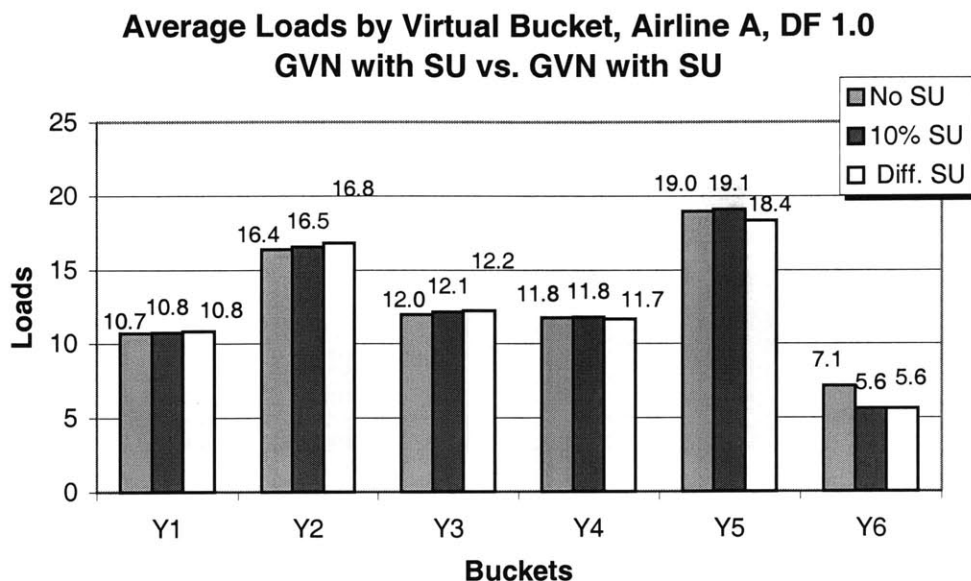
Table 32: Percentage Gains as Function of Demand Factor and Input Sell-up Rates, GVN vs. GVN

Demand Factor	Input Sell-up Rates	Percentage Revenue Gain Over Base Case	
		Both Airlines Account for Sell-up	Airline A only Accounts for Sell-up
0.8	0%-0%	0.09%	0.09%
	10%-10%	0.19%	0.35%
	Both Differential	0.28%	0.56%
1	0%-0%	0.50%	0.50%
	10%-10%	0.65%	0.95%
	Both Differential	0.97%	1.58%
1.2	0%-0%	0.42%	0.42%
	10%-10%	0.80%	1.20%
	Both Differential	2.27%	2.51%

Table 32 shows that the revenue gains are, as usual, highest when the airlines use differential input sell-up rates. It is also apparent in this table that the revenue gains achieved at differential input sell-up rates are higher than when no sell-up is assumed. This points out that there is a gain to be achieved by accounting for sell-up in a virtual bucket environment. Accounting for sell-up with differential input sell-up rates roughly doubles the gains obtained by switching to GVN from EMSRb at demand factor 1.0. At demand factors 0.8 and 1.2, the difference is even greater, more than three times greater at demand factor 0.8 and almost six times greater at demand factor 1.2. Table 32 also shows us that the revenue gains are lower when both airlines account for sell-up than when only airline A does. Revenues drop by about 0.3% at demand factor 0.8 and 1.2 and by more than 0.6% at demand factor 1.0 and at differential input sell-up rates.

In terms of loads, we see in Figure 33 the distribution of loads amongst the six virtual buckets as a function of the input sell-up rates. As previously observed, our input differential sell-up rates consistently achieve the highest loads in the first three buckets, while in the last three buckets, the loads are higher in the other cases. Moreover, the average leg load factor is very similar in both cases. The revenue difference can therefore be explained mostly by the difference in the distribution of passengers across virtual buckets. This distribution favors the set of differential sell-up rates, as they lead to more high fare passengers, on average.

Figure 33: Loads by Virtual Buckets as a Function of Input Sell-up Rates, DF 1.0, GVN vs. GVN



In summary, our results again show that accounting for sell-up, even when both airlines do so, does indeed bring revenue gains to the airline taking this possibility into consideration. The gains range from 0.6% to 2.5% depending on the demand factor. Moreover, as usual, and as suggested by **Bohutinsky**²⁰, the revenue gains reach a peak at differential input sell-up rates.

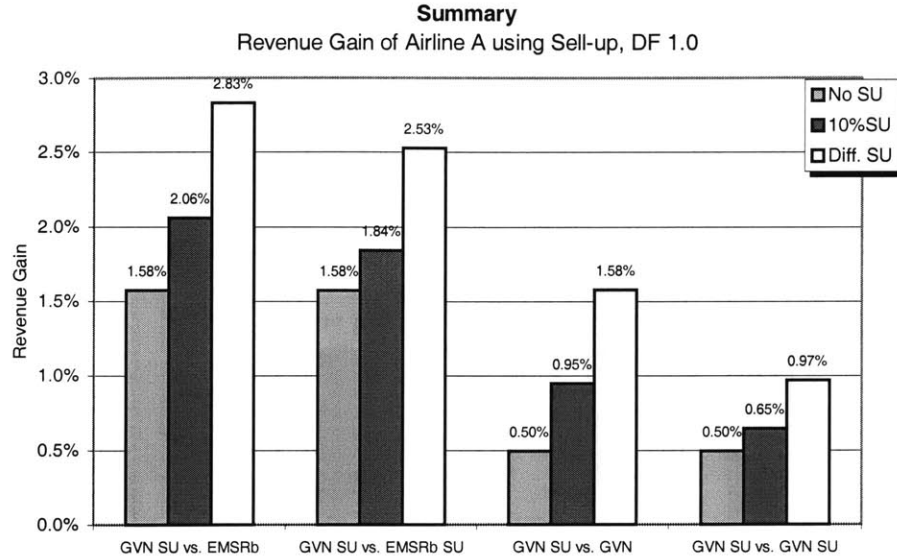
5.2.3 Summary

In this section, we studied the effect of sell-up models on virtual buckets, that is, when one or both airlines use Greedy Virtual Nesting as their Revenue Management algorithm. In addition, we were only looking at the effect of accounting for sell-up when the detruncation method used is Booking Curve detruncation. Our findings show that once again, accounting for sell-up increases the protection for higher classes and increases the revenue gains, as long as the input sell-up rate does not lead to overprotection.

Moreover, our results also show, as was the case with EMSRb, that when both airlines account for sell-up, the revenue gains are lower than when only one airline accounts for sell-up, for the airline accounting for sell-up.

Finally, at this point we also note that GVN benefits more from accounting for sell-up when its competitor is using FCYM, in the network 6B scenario. We will discuss at the end of this chapter why network 6B favors GVN over EMSRb, and, in our last chapter we will be testing the influence of a larger network on these results. This will give us a sense of the impact of the network definition on revenue gains. Figure 34 summarizes our previous results at demand factor 1.0.

Figure 34: Summary of Booking Curve Detruncation Results at Demand Factor 1.0



5.3 Projection Detruncation

In this section, we now allow airline A to modify its detruncation method in an attempt to further increase its revenues. Airline A will still be accounting for sell-up while airline B will either take this possibility into consideration or not.

This section will be divided into two subsections. In the first subsection, we will study in details the impact of Projection detruncation on the gains achieved by GVN combined with sell-up models. The competing airline will, in that case, be using EMSRb FCYM combined with Booking Curve detruncation, but without accounting for sell-up. We will then study the changes in revenue due to Projection detruncation and compare them to revenues obtained with Booking Curve detruncation. In the second subsection, we will then focus on more general results, by changing the competitor's Revenue Management scheme: We will be modifying the detruncation, the algorithm used and accounting for sell-up in some cases and not in other cases.

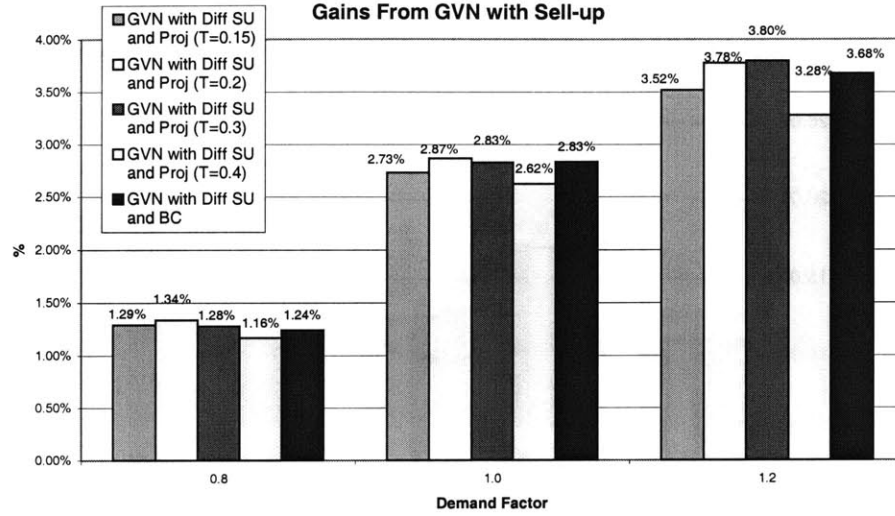
5.3.1 GVN with Projection Detruncation vs. Base Case EMSRb

In this subsection, we explain the results obtained through the simulation of airline A using GVN combined with the sell-up heuristic and Projection detruncation, when the competitor uses EMSRb with Booking Curve detruncation but without sell-up. We will be looking at various τ values, as the reader will remember from previous analyses (c.f. Zickus⁹, Chapter 4.2.2.a) that the τ value is directly linked to the aggressiveness of the detruncation.

5.3.1.a Revenues

Figure 35 shows the revenue gains of airline A depending on the τ value and on the demand factor, for airline A using GVN with sell-up and airline B using EMSRb without sell-up.

Figure 35: Airline A Revenue Gains as a Function of τ , GVN with Differential Sell-up Assumed vs. EMSRb w/o Sell-up



It appears clearly that there is a peak revenue gain achieved somewhere between $\tau=0.2$ and 0.3 . At $\tau<0.2$, we see from this chart that revenues drop, indicating that the detruncation has become too aggressive and that airline A is overprotecting, while for $\tau>0.3$, revenues also drop, but this now indicates that airline A is not protecting enough. We will confirm these statements in the next paragraphs when we look at loads.

These results also show clearly that the additional gain to be expected from Projection detruncation is in the neighborhood of 0.1% at all demand factors, that is, very low.

Hence, our first important two results here are the following:

- Projection detruncation with $0.2<\tau<0.3$ can bring about 0.1% additional revenues to the airline accounting for sell-up and using GVN.
- The “optimal” τ value is very similar to the “optimal” previously obtained with EMSRb.

5.3.1.b Loads

We will now be looking at loads by virtual bucket. Figure 36 details the distribution of loads as a function of the detruncation method (τ value or Booking Curve detruncation).

Figure 36: Loads by Virtual Bucket, GVN with Differential Sell-up Rates Assumed and Projection Detruncation vs. EMSRb w/o Sell-up and Booking Curve Detruncation, DF 1.0

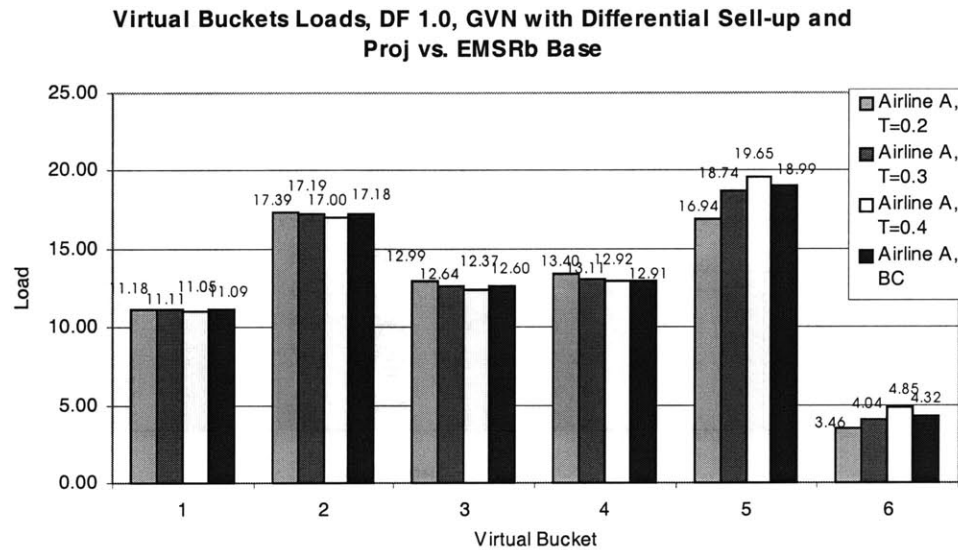


Figure 36 confirms what we stated earlier. Indeed, we see on Figure 36 that at $\tau=0.2$, the loads are higher than in any other case in the first four buckets, while substantially lower than all other three cases in the lower two buckets. Conversely, the exact opposite is observed at $\tau=0.4$. Indeed, in this case, the loads are the lowest in the first four buckets but the highest in the last two. This confirms that at $\tau < 0.2$ or $\tau > 0.3$ we are respectively over- and under-protecting.

Figure 36 also shows that the loads at $\tau=0.3$ are very similar to those of Booking Curve detruncation, at demand factor 1.0, which corresponds to the fact that revenue gains are very similar whether we use Projection detruncation with $\tau=0.3$ or Booking Curve detruncation.

Finally, Figure 36 shows us the distribution of absolute loads amongst the virtual buckets at $\tau=0.2$, as compared to Booking Curve detruncation. It turns out that the major difference with Booking Curve detruncation loads is that the loads in the first four buckets are higher, while the loads in the last two are lower. Overall, the average leg load factor is about 75% at $\tau=0.2$ and demand factor 1.0, while it is about 77% with Booking Curve detruncation, at the same demand factor. This leads us to the conclusion that the 0.1% difference in revenues between these two cases is due to the fact that Projection detruncation allows the airline to achieve higher loads

in the high revenue buckets. Moreover, these higher loads more than offset the revenue losses suffered in the lowest two buckets, where the loads are higher for Booking Curve detruncation.

In summary, these results show that there are revenue gains to be obtained by switching to Projection detruncation, and that these additional revenue gains are very similar to those obtained with EMSRb. The “optimal” τ value lies within the interval [0.2; 0.3]. However, we also noted here that these additional revenue gains are very low, in the vicinity of 0.1%, and therefore, once again, minimal. Moreover, these gains are very sensitive to the change in the τ value and all the more so as the demand factor increases. Indeed, at $\tau=0.15$ and demand factor 0.8, the overall revenue gains are still above Booking Curve detruncation revenue gains, but below $\tau=0.2$ revenue gains. However, at demand factors 1.0 and 1.2, the gap increases and the revenue gains drop below those of Booking Curve detruncation at $\tau=0.15$.

5.3.2 Effect of the Competitor's Revenue Management Method

In this section, we allow the competing airline to upgrade its Revenue Management system to various levels of “sophistication”. We think it is necessary here to remind the reader that when we refer to differential input sell-up rates, we always imply the following:

- In the case of an airline using GVN, the input sell-up rates are 40%-35%-30%-20% and 10% by virtual bucket, from top down.
- In the case of an airline using FCYM, the input sell-up rates are 45%-30% and 20% by fare class, from top down.

In all the cases studied below, we will always have the simulations set up so that either only one airline accounts for sell-up, or when both airlines do, they will be using the “same” input sell-up rates. By this we mean that if they are both using constant input sell-up rates, they will be using the same input sell-up rates by virtual buckets or fare class. If they are both using differential input sell-up rates, they will be using the above-described rates.

Finally, the purpose of 5.3 being to evaluate the impact of Projection detruncation combined with GVN and the **Belobaba-Weatherford** Sell-up model²² on the revenues of airline A, airline A will obviously always be using GVN with Projection detruncation and sell-up.

In the following paragraphs we describe the simulations we ran and then discuss our results.

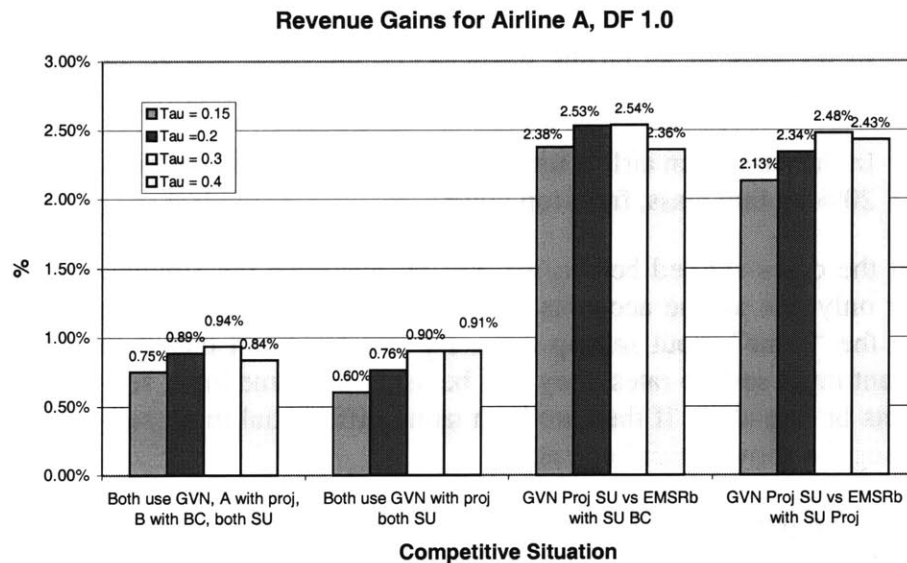
5.3.2.a Simulations

At various τ values we ran the following four sets of simulations:

- Both airlines use GVN combined with the sell-up model, while only airline A uses Projection detruncation. We vary the τ value in order to get the best possible results in terms of revenues.
- Both airlines use the same combination of GVN, input sell-up rates and Projection detruncation.
- Airline A uses GVN with Projection detruncation and input sell-up, while airline B uses FCYM (EMSRb) combined with sell-up and Projection detruncation.
- Finally, both airlines use Projection detruncation combined with sell-up, but airline A uses GVN as its Revenue Management algorithm while airline B uses EMSRb FCYM.

Figure 37 summarizes the revenue gains for airline A in all of the above cases as a function of the τ value. The reader should remember that all revenue gains are against the “base case”.

Figure 37: Summary of Revenue Gains for all Cases as a Function of τ , DF 1.0

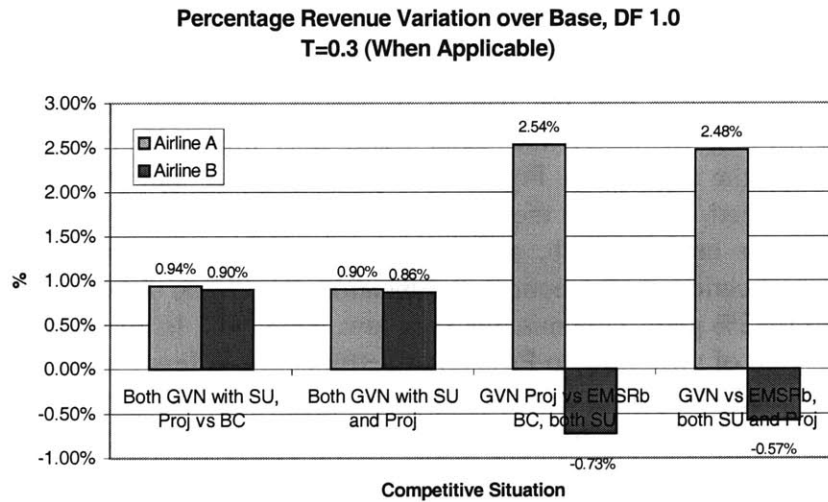


Our first comment on these results is that the revenues seem to reach a peak once again at $\tau \approx 0.3$. This is similar to the previously observed results. Let us now compare these results to the gains achieved with Booking Curve detruncation, on a case-by-case basis.

First of all, the reader will recall that at demand factor 1.0, GVN vs. GVN, both with sell-up and booking curve detruncation yielded revenue gains of 0.97% for both airlines. We therefore note that this gain is higher than what we achieve here with

Projection detruncation. The revenue gains are nonetheless very close to the 0.97% gains observed previously. However, it seems from Figure 38 that switching to Projection detruncation for one of the two airlines leads to an overall decrease in revenues for both airlines. Indeed, Figure 38 shows that the revenues for airline B have decreased slightly to 0.9%. This could be explained by an inadequate protection level that airline A creates with Projection detruncation. This inadequate protection causes airline A's revenues to drop slightly, but also airline B's revenues to decrease. Indeed, as the protection changes for airline A, airline B's loads drop slightly. Airline A is protecting a little more, therefore airline B gets more of the low yield traffic and its revenues decrease. This is shown by the fact that airline A's revenues decrease at $\tau=0.4$. Overall, in this case, there seems to be no revenue to be gained from switching to Projection detruncation for airline A.

Figure 38: Revenue Gains for Both Airlines in all Four Cases, DF 1.0



Our second case assumes that both airlines use the same Revenue Management scheme, that is, both use GVN with sell-up and Projection detruncation. Once again, there is a peak in revenues at $\tau \approx 0.3$. However, once more, the maximum revenue gain seems to be about 0.9%, which are below the revenue gains when the detruncation method used was Booking Curve detruncation. This indicates that in the case when both airlines account for sell-up, Projection detruncation leads to lower revenue gains than Booking Curve detruncation. This is independent of the τ value.

Our third case assumes that airline A uses GVN with sell-up and Projection detruncation, while its competitor uses EMSRb with sell-up and Booking Curve detruncation. Once again, we remind the reader that the revenue gains when airline A used Booking Curve detruncation instead of Projection detruncation in this same situation was 2.53% at demand factor 1.0. We now have a revenue gain of 2.54% at $\tau=0.3$ and at demand factor 1.0. This revenue gain, as compared to our base case, is very close to the revenue gain achieved with Booking Curve detruncation. Both

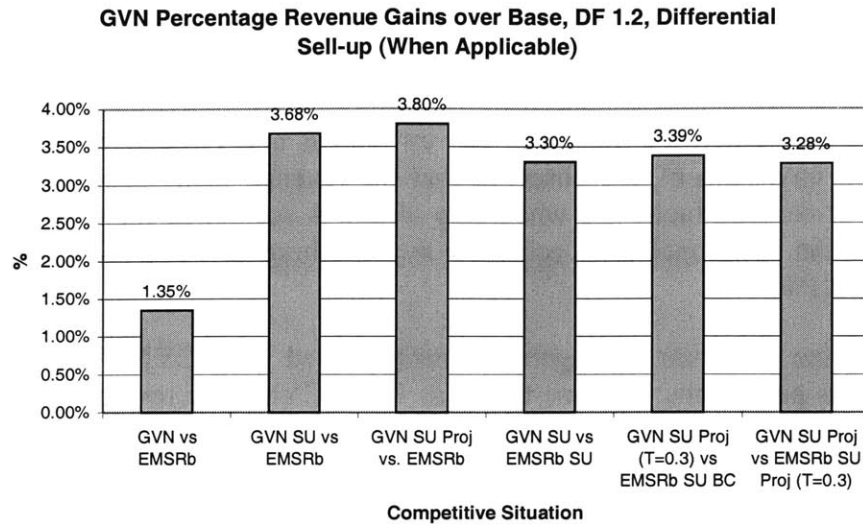
revenue gains are so close that we cannot attribute this revenue increase to Projection detruncation and be sure that it is not due to the randomness of the simulation.

Finally, our last case opposes airline A with GVN to airline B with EMSRb, both accounting for sell-up and using Projection detruncation. Once again, we see on that the revenue gains are lower than they were in the case when neither airline was using Projection detruncation. Moreover, we also observe that the revenue gains decreased when compared to the previous case when only airline A was using Projection detruncation. We also note that airline B's losses were 0.67% at demand factor 1.0 when both airlines accounted for sell-up and neither used Projection detruncation. Here, the losses of airline B have decreased to 0.57% (c.f. Figure 38). This tends to confirm the fact that airline A is less efficient at protecting seats for high revenue passengers and that airline B regained some of its lost traffic.

Overall, all these four cases confirm what our initial results tended to prove, that is, that there is very little, if anything, to be gained from switching to Projection detruncation, when sell-up is being accounted for.

In order to confirm that the pattern of results described above holds at higher demand factors, we tested the same cases at a higher demand factor (1.2) and summarized the results on Figure 39. This confirms that the revenue gains are minimal. Indeed, if we take the example of airline A using GVN with sell-up when its competitor uses EMSRb, and switch the detruncation method from Booking Curve detruncation to Projection detruncation for airline A alone, we see that we achieve a 0.12% gain. This remains very low. Moreover, in all the other cases shown here, the gain of switching to Projection detruncation is less than 0.1%, when sell-up rates are used. This is a critical difference with **Zickus's** results⁹ on Projection detruncation. The reason for this much lower impact of the detruncation method on the revenue gains is that accounting for sell-up and using a more aggressive detruncation method have very similar effects on protection levels. However, accounting for sell-up allows for finer modifications of these protections. Therefore, the additional protection reached with Projection detruncation becomes unnecessary.

Figure 39: Revenue Gains as a Function of the Competitive Situation, DF 1.2



5.3.3 Summary

In this section we discussed the impact of Projection detruncation on the revenues achieved by an airline using GVN as its Revenue Management method combined with the **Belobaba-Weatherford** Sell-up heuristic²². Our results show that the additional gains from Projection detruncation are in the neighborhood of 0.1% or less at $\tau=0.3$. Our results clearly show that there is an optimal τ value between 0.2 and 0.3, as was the case in the EMSRb case. Overall, our conclusion is that the detruncation method, in a virtual bucket environment, has very little impact on the airline's revenue gains when the airline also accounts for sell-up. We explained in 5.3.2.a why the detruncation method had a smaller impact on revenues than accounting for sell-up. Compared to the gains achieved by accounting for sell-up, the incremental gains achieved through Projection detruncation are very small.

In this paragraph, we would also like to stress the fact that Network 6B, our simulated network is extremely "GVN-friendly". Indeed, we explained earlier in the thesis that GVN performs much better in an environment where connecting demand is high and local demand is low and where short-haul legs have higher demand than long hauls, combined with a fare structure that has connecting passengers pay higher fares than local passengers. It turns out that Network 6B combines both of these properties in that it was set up to have 60 percent connecting passengers and a suitable fare structure.

Overall, network 6B clearly favors GVN over EMSRb, which is one factor that should be kept in mind regarding the comparison of EMSRb and GVN gains.

5.4 Conclusion: Sell-up vs. Detruncation - Present Findings

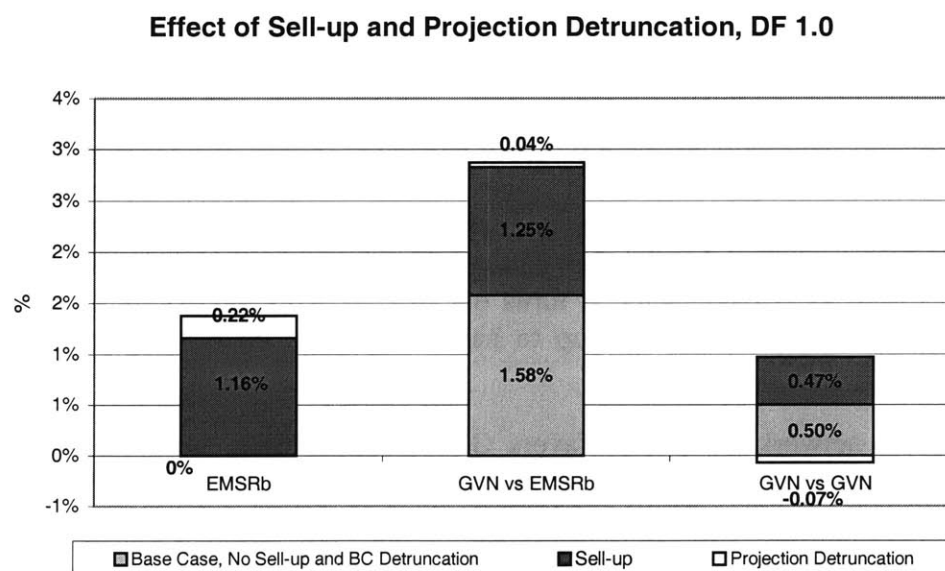
At this point of the thesis, we have looked at the effect of accounting for sell-up and at the impact of the detruncation method. We think that it is important at this point to summarize our findings and draw preliminary conclusions.

When we studied the impact of sell-up models and Projection detruncation in the FCYM environment, we observed that the revenue gains could reach as much as 1.5% at demand factor 1.2 when only airline A accounted for sell-up. In all FCYM cases, the additional gain achieved by switching to Projection detruncation was about 0.1%.

When the Revenue Management method used was GVN, we observed in the previous paragraphs the same trends as with FCYM. The revenue gains achieved by accounting for sell-up are, however, considerably higher than those obtained by changing the detruncation method. Once more, we refer the reader to **Zickus**⁹, who explained in detail the influence of Projection detruncation on the revenue gains of airline A. The gains were about 0.7% at demand factor 1.0 and $\tau=0.15$, while we achieved higher revenue gains by accounting for sell-up in a fare class Revenue Management environment. Moreover, this detruncation method adds one more parameter to the simulator and makes it more complicated to differentiate all the effects influencing the revenue variations.

Overall, as shown on Figure 40, the incremental gains achieved through sell-up models are consistently higher than those of Projection detruncation. However, they depend on the competitive situation.

Figure 40: Impact of Sell-up Models and Projection Detruncation at Demand Factor 1.0



In this chapter, we then conclude that the effect of accounting for sell-up is the overriding effect when it comes to increasing GVN revenue gains.

Our final comment at this stage is that **Bohutinsky's** thesis²⁰ regarding the fact that differential input sell-up rates better represent the passengers' behavior than constant input sell-up rates seems to be confirmed by our results. Indeed, we consistently get higher revenue gains with these differential input sell-up rates, compared to with constant input sell-up rates.

Chapter 6: Comparison with Other O-D Methods – Validation of our Results in a Larger Network

In this chapter, we will compare the results we obtained by accounting for sell-up and modifying the detruncation method to previously described results for more advanced O-D methods. In particular, we will be comparing our revenue gains to those of the Bid Price methods described in Part I, as well as those obtained with DAVN. These results were established by Lee⁸.

This chapter will also allow us to describe the results observed when accounting for sell-up in a bigger network. As we previously explained, we would like to validate our previous sell-up results in this larger network. In particular, we wish to study the impact of the network size and layout on the revenue gains.

6.1 Comparison with Other O-D Methods

The reader will recall from previous chapters that the revenue gains from accounting for sell-up greatly outweigh those achieved by switching to Projection detruncation when the airline is accounting for sell-up: We explained in Part II, Chapter 5.4, that switching to Projection detruncation led to 0.1% to 0.2% incremental revenue gains. When not accounting for sell-up, Zickus⁹ showed that the revenue gains achieved by switching to Projection detruncation are 0.55% at demand factor 1.0, and therefore comparable to or lower than those achieved by accounting for sell-up, depending on the case. The reader will recall that the revenue gains at demand factor 1.0 when only airline A was accounting for sell-up (with FCYM and Booking Curve detruncation) was 1.18%. At the same demand factor, but when both airlines were accounting for sell-up and inputting differential sell-up rates, the revenue gains dropped to 0.72%. At demand factor 1.0, the percentage revenue gains are the following:

- 1.5% when only airline A accounts for sell-up in a FCYM environment,
- 0.7% when both airlines account for sell-up in a FCYM environment,
- 2.8% when airline A accounts for sell-up in a virtual bucket environment while airline B uses FCYM without accounting for sell-up,
- 2.5% in the same situation, but when both airlines account for sell-up,
- 1.5% when only airline A accounts for sell-up but both airlines use GVN,
- 1% when both airlines account for sell-up in a virtual bucket environment.

All these results are with Booking Curve detruncation, as we established that the detruncation method had a minor additional effect on the revenue gains.

Our purpose in this section will then be to compare these results to those obtained with other O-D methods, namely, Netbid, HBP, DAVN and ProBP, all described in Lee's thesis⁸. We will base our comparison on revenue gains over our base case.

6.1.1 Previous Results

Lee⁸ and Bratu¹⁷ established the following results on Network 6B. In particular, the revenue gains over the base case are shown on Figure 41.

Figure 41: Revenue Gains over Base as a Function of the Competitive Situation, DF 1.0, Network 6B

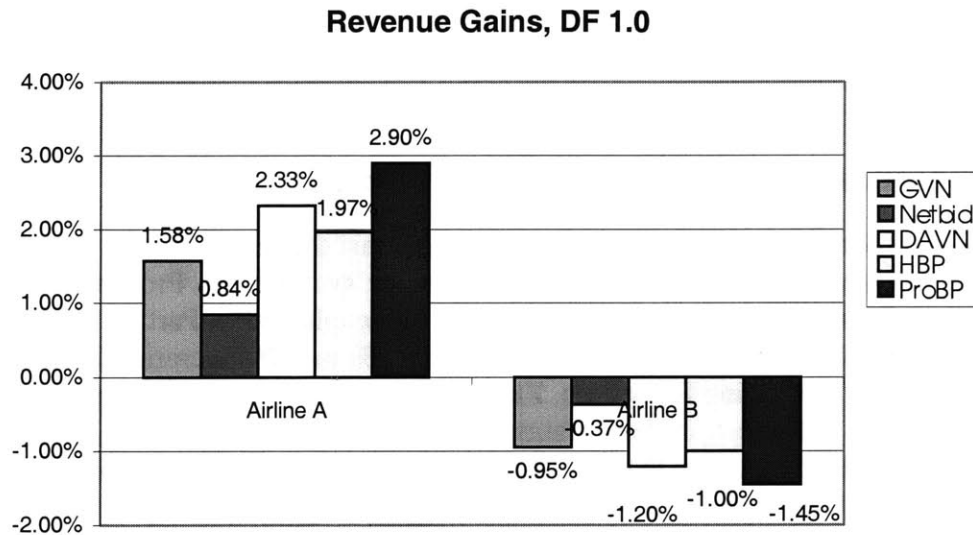


Figure 41 shows that the highest revenue generating Revenue Management methods are DAVN and ProBP. The reader is referred to Part I of this thesis for more detailed explanations of both of these methods. The major characteristics of these two methods are respectively that DAVN uses a displacement cost to evaluate the “network” revenue of having a passenger book a given flight, while ProBP calculates the bid price for each leg according to a proration system that incorporates both stochasticity of demands and implicit nesting of ODFs over the network.

All these results were obtained on Network 6B and can therefore safely be compared to our previous results. To simplify our comparisons, we will only focus on the gains achieved by accounting for sell-up.

6.1.2 Comparison with our Results

In this section, we present our results and previously established results by **Lee**⁸ and **Bratu**¹³. All our results involving Sell-up will be at differential input sell-up rates, as we confirmed **Bohutinsky**'s thesis²⁰ that differential input sell-up rates lead to higher revenue gains.

6.1.2.a Comparison with Earlier Results

Figure 42: Revenue Gains as a Function of the Competitive Situation, DF 1.0

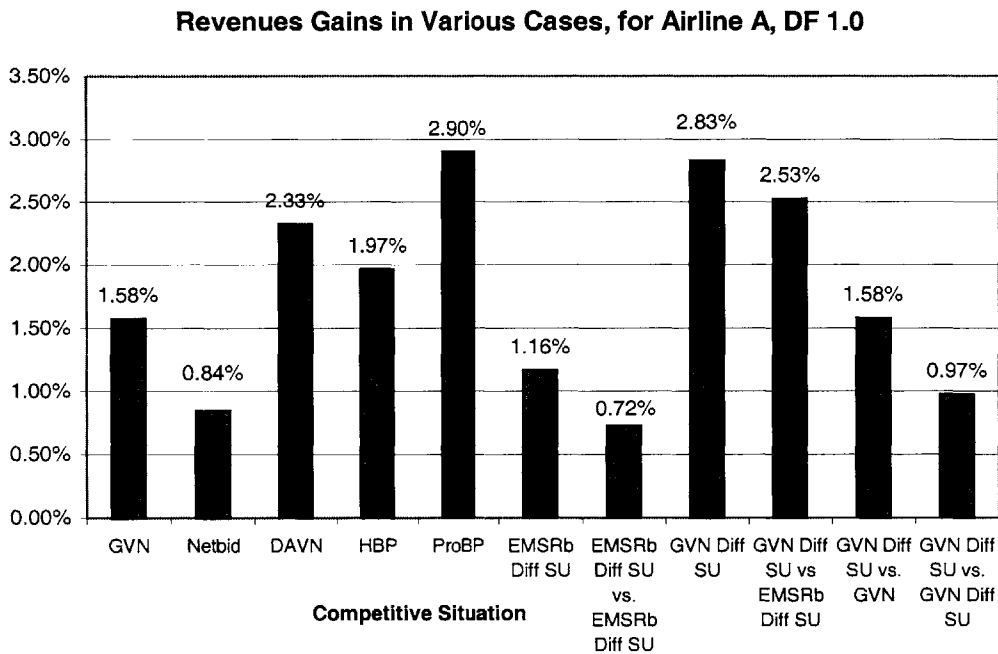


Figure 42 is essentially divided into two parts. The left-hand side of the chart reports the results described by **Lee**⁸ and presented in the previous paragraph. The right-hand side of the chart summarizes all our previous results at demand factor 1.0 and with Booking Curve detruncation.

First of all, it appears that EMSRb benefits from accounting for sell-up and achieves revenue gains comparable to those of Netbid. The reader is referred to the Part I of this thesis for more information on Netbid. We remind the reader that Netbid solves a deterministic LP to obtain the bid prices for each leg. Whether or not the competitor accounts for sell-up, the revenue gains from accounting for sell-up in a FCYM environment are equivalent to those of Netbid. However, the only relevant comparison that we can make here concerns the case when the competitor does not account for sell-up. As we already stated, EMSRb then does better than Netbid, but does not quite achieve the same gains as GVN. There remains a difference of about 0.4%.

The second major result that appears on this chart is the very good performance of GVN with sell-up when the competitor uses FCYM, be it with or without sell-up. In terms of revenue gains, GVN with sell-up always outperforms GVN, when its competitor uses FCYM. Moreover, the revenue gains get very close to those of ProBP. Once again, the reader can refer to the first part of this thesis for more information on both of these Revenue Management methods. We can infer that revenue gains similar to those of the “best” O-D methods can be achieved through “lower technology” Revenue Management methods such as GVN.

Finally, we also observe on Figure 42 that regardless of the competitive situation, when airline A uses GVN combined with the sell-up heuristic, its revenue gains are always positive and always better than those of Netbid.

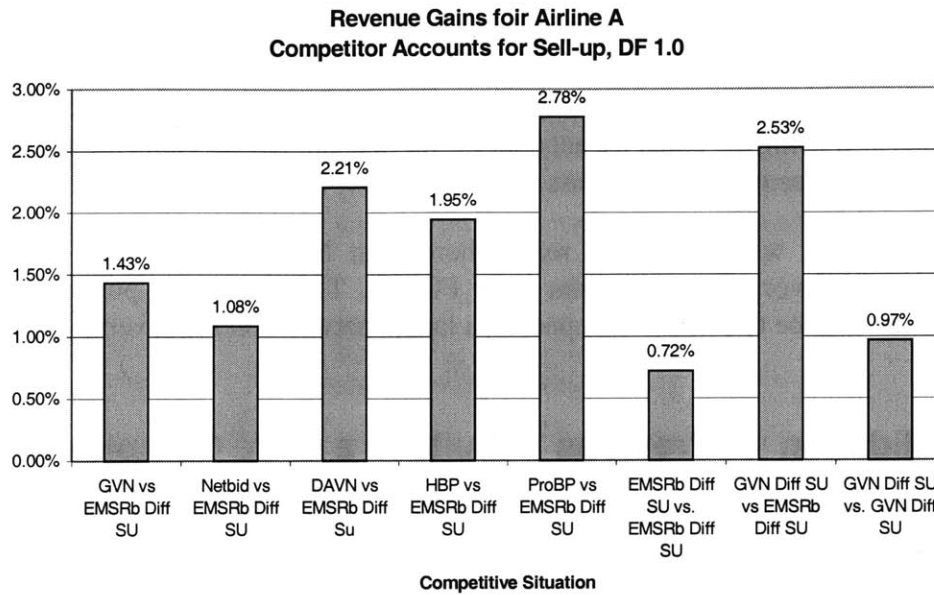
Overall, we observe that the revenues obtained by accounting for sell-up are generally good. In particular, when airline A uses GVN, it achieves revenues above those of ProBP. Overall, the gain over EMSRb is close to 3 percent.

Our conclusion to this first section is that the revenue gains achieved by accounting for sell-up, and in particular in a virtual bucket environment, are very high and can be higher than those of the “best” O-D Revenue Management methods tested to date, when they do not account for sell-up.

6.1.2.b Effect of Competitor Using Sell-up on O-D Methods

In this section, we now focus on the effect of accounting for sell-up when the competitor also accounts for sell-up, and compare the results to those of all the other O-D methods when the competitor accounts for sell-up. In Figure 43, we present the revenue gains (compared to the base case EMSRb vs. EMSRb) for airline A when airline B accounts for sell-up. Our purpose in this section will be to compare the performance of the O-D methods when the competitor accounts for sell-up to the revenue gains achieved in similar conditions by EMSRb accounting for sell-up and GVN accounting for sell-up.

Figure 43: Revenue Gains for Airline A when Competitor Accounts for Sell-up, DF 1.0



The most striking results in this case are, first of all, that GVN with sell-up does substantially better than GVN without sell-up (when the competitor is accounting for sell-up in a FCYM environment): It achieves revenues almost twice as high as it did without accounting for sell-up (when airline B is accounting for sell-up). Second of all, GVN with sell-up performs very well and has the second best revenue gains, just below ProBP.

EMSRb, on the other hand, does not do quite as well. The revenue gains prove that we are still doing better than when no sell-up is assumed. However, any other Revenue Management method is performing better than EMSRb in this case.

We also note here that Netbid is the only Revenue Management method that actually performs better when its competitor is accounting for sell-up (c.f. Figure 42 and Figure 43). This is most likely related to the fact that airline B now protects more and therefore spills passengers towards the airline A, which under Netbid, accepts these passengers.

6.1.3 Conclusion

These results suggest that accounting for sell-up, either in a Fare Class or Virtual Bucket environment, brings substantial revenue gains to the airline accounting for sell-up. Moreover, in the case of GVN, we observe that these gains are equivalent to those of ProBP without sell-up, our former best case.

When the competitor is not accounting for sell-up, EMSRb and GVN, when combined with the sell-up heuristic, lead to substantial revenue increases that respectively compare to Netbid and ProBP revenue gains.

When the competitor is accounting for sell-up, GVN does very well, as opposed to all the other O-D methods, and has revenue gains comparable to those of ProBP (the highest revenue generating Revenue Management method). EMSRb, on the other hand, still achieves gains by accounting for sell-up, but its gains are the lowest of all methods presented on our charts.

Once again, we remind the reader here, that Network 6B is GVN friendly and therefore favors this algorithm over FCYM. Therefore, the purpose of our next section will be to study the impact of a larger network on the revenue gains.

6.2 Validation of Previous Results in a more Complex Network

In this section we will be running our simulations on the bigger network C. Our goal will be to confirm or reject our results on this bigger network. In particular, we will be focusing on GVN and how it performs in an environment that is less well suited to its “greedy” preference for connecting passengers.

The first subsection will describe Network C and its specificity. The following subsection will summarize and explain our results.

6.2.1 Network C

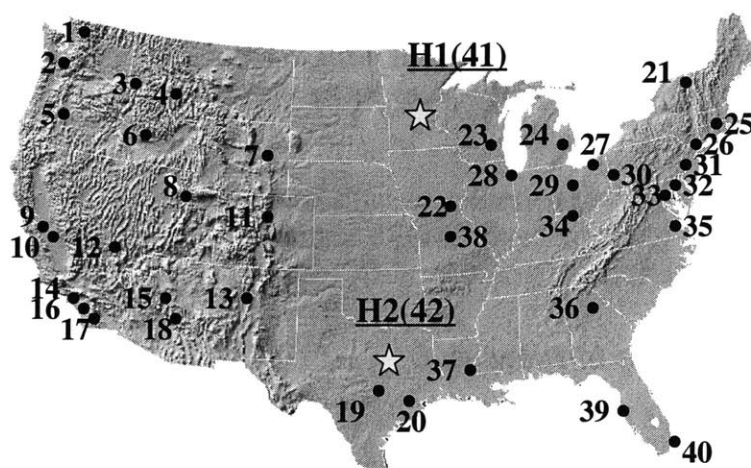
Let us first introduce our new network. Once more, we remind the reader here that we introduce this network with the sole purpose of validating the results we observed on the smaller Network 6B. The major difference with Network 6B is the size of the network.

In particular, Network C has 40 spoke cities with two hubs, one for each of the two competing airlines. This network was designed to better reflect the US geography. Therefore, the cities were set up with twenty on each side of the hubs, the hubs being located in the “middle” of the network, as shown on the following figure. Moreover, this network was generated using demand indices and attractiveness of cities

The flows on this network are assumed to be unidirectional, from West to East, with three different banks (a bank is a time period during which a set of flights arrives from the spoke cities in time to connect with flights departing from the hub). In this case, 21 flights arrive at the hub and depart from the hub during each bank.

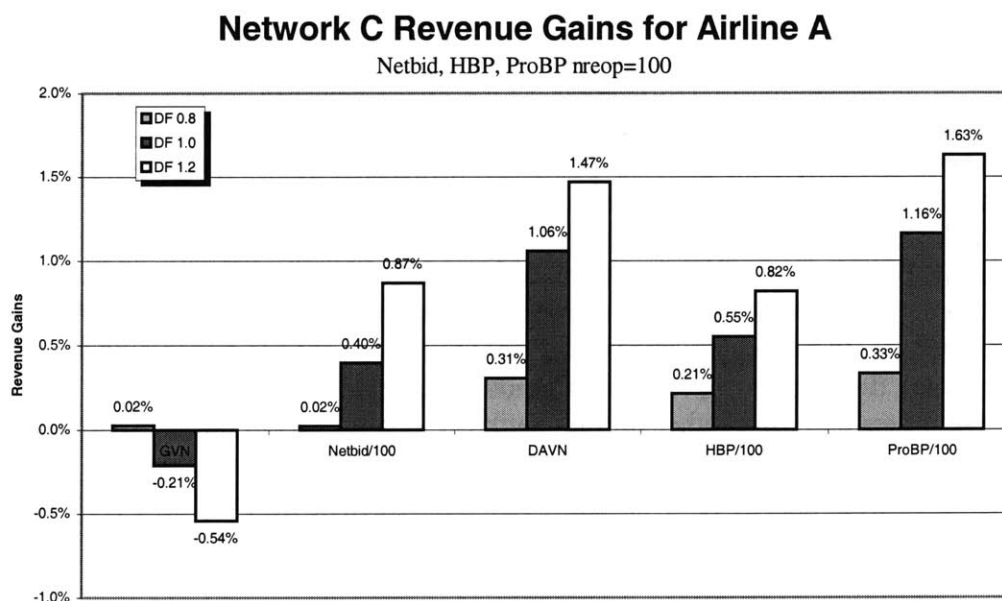
Overall, this network has 252 flight legs per day that serve a total of 482 O-D markets. The reader will recall here that Network 6B, in comparison, had only 24 legs that served a total of 54 markets.

Figure 44: Network C Geographical Layout



The revenue gains for the base cases were run on this network in order to get an idea of the new revenues on this bigger network. The following chart shows the revenue gains at all three demand factors (0.8, 1.0 and 1.2).

Figure 45: Network C Revenue Gains over Base Case (EMSRb vs. EMSRb)



These results (Figure 45) show that GVN now performs much worse than any other Revenue Management method, including EMSRb. The reason for this is that the network is not GVN friendly anymore. It was setup to allocate demand to all legs in a more random manner. However, the assignment of demand results in non-

symmetric distribution of demand as a function of distance, unlike Network 6B. Overall, we end up with a less “virtual-friendly” network.

The pattern of results, other than the fact that GVN does not perform well, has remained the same in that DAVN still leads to the second highest revenue gains after those of ProBP. However, HBP does better than Netbid only at demand factors 0.8 and 1.0.

Having reviewed the base results, we will now focus on the effect of sell-up models on the revenue gains for both EMSRb and GVN in particular, once again without focusing on the effect of the detruncation method.

6.2.2 Sell-up Results in Network C

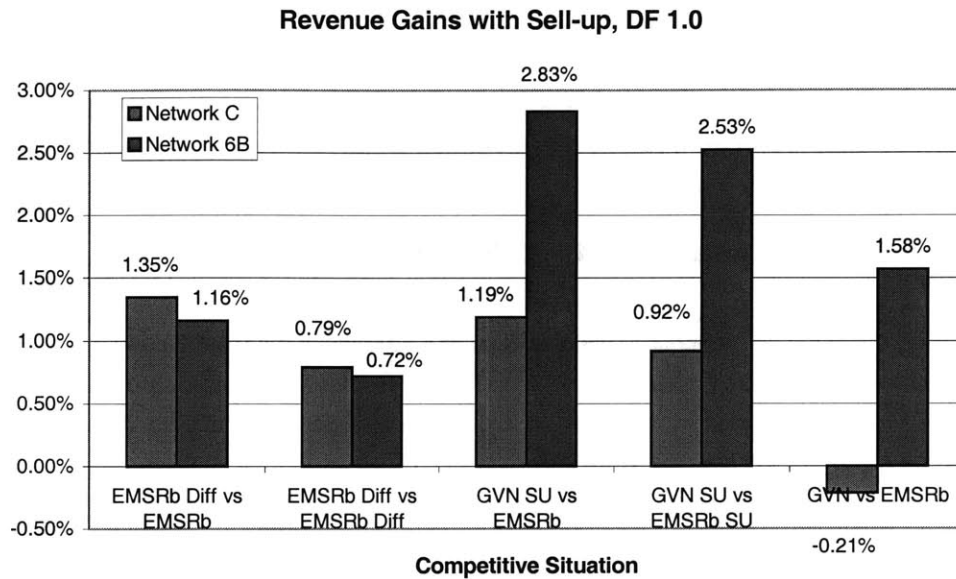
In this section, we describe the revenue gains achieved by accounting for sell-up in a fare class environment and in a virtual bucket environment. As stated earlier, we would like to confirm or reject our previous findings, that is, that accounting for sell-up leads to substantial revenue gains either in a fare class or a virtual bucket environment.

In the following paragraphs, we will be looking at the percentage revenue gains to evaluate whether the sell-up related gains are still substantial. We will compare these percentage gains to previously observed gains on Network 6B to see how they change in this larger network.

6.2.3 Influence of Network Size on EMSRb and GVN

We have already shown on the previous graphs (Figure 43 and Figure 45), that the revenue gains have already dropped by switching from Network 6B to Network C for all Revenue Management methods. In terms of revenue gains for EMSRb and GVN, Figure 46 shows the trends at demand factor 1.0 on this larger network. It should be stressed here that there are major differences in terms of load factors between networks 6B and C, differences that are responsible for much of the revenue differences. In particular, Network C has lower load factors due to a wider (and more realistic) distribution of demands among legs. However, we are not focusing on these issues here, but rather on the “ranking” changes from one network to the next. That is, how does GVN perform compared to EMSRb and how did the ranking change from what we observed in Network 6B. The other major issue we are interested in is the revenue gains attributable to sell-up models.

Figure 46: Comparison between Network 6B and Network C Revenues, EMSRb and GVN, DF 1.0



On this larger network, two major effects appear on the chart:

- First of all, we observe that EMSRb is now doing a little better in this larger network, regardless of whether the competitor is accounting for sell-up or not.
- Second, we also note that GVN is doing considerably worse than it used to in the smaller network. In particular, when no sell-up is assumed, GVN actually incurs **losses**. However, once we account for sell-up, it generates revenues and even outperforms EMSRb when both airlines account for sell-up.

Overall, we see that the non-friendliness of the network to virtual buckets clearly reflects on the ranking of the Revenue Management methods : GVN now does worse than EMSRb (when neither airline is accounting for sell-up, and in most cases involving sell-up), while it did much better than EMSRb in network 6B. When only airline A is accounting for sell-up, EMSRb leads to greater revenue gains than GVN, both methods having positive revenue gains. However, when both competitors account for sell-up, then GVN outperforms EMSRb by 0.1%, as can be seen on Figure 46.

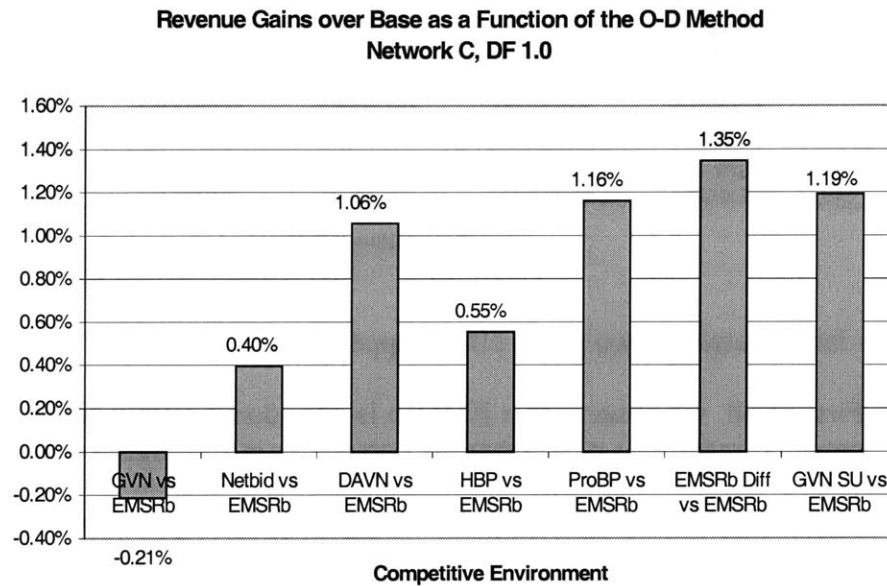
This first set of results allows us to conclude that the network size and layout (and especially load factors) do play an important role on revenues, but that accounting for sell-up still brings substantial revenue gains. These revenue gains (over the base case) have decreased by a substantial order of magnitude for GVN, but have remained in the same neighborhood for EMSRb. Finally, our last comment on this set of results is that GVN still brings slightly higher revenues than EMSRb, when both airlines account for sell-up with differential rates. Once again, we need to stress

the fact that the two networks have substantial load differences at demand factor 1.0 which are in part responsible for the differences in revenue gains observed between the two networks.

6.2.4 Comparison with other O-D Methods

In the following paragraphs, we now present the revenue gains of all the other O-D methods and compare them to the revenues achieved by accounting for sell-up, both in the EMSRb and the GVN case.

Figure 47: Revenue Gains for all O-D Methods, DF 1.0



The first major result shown on Figure 47 is the dramatic change in the revenue gains of all five O-D methods: GVN, Netbid, DAVN, HBP and ProBP. These revenue gains have dropped substantially from the level they reached in Network 6B. However, once again, this is attributable to the differences in load factors between the two airlines. However, in terms of “ranking” of the methods, ProBP is still the Revenue Management method that leads to the highest revenue gains (without sell-up) with 1.16% gains at demand factor 1.0. In addition, as we described earlier, GVN suffers most from the change of network with a loss of about 1.8% revenues at demand factor 1.0, leading to overall losses rather than revenue gains, as shown on Figure 46.

When airline A accounts for sell-up, at differential input sell-up rates, we observe that the highest revenue gains are now achieved with EMSRb. Unlike was the case with Network 6B, EMSRb with sell-up now clearly outperforms all O-D methods (as shown on Figure 47). Moreover, the revenue gains from GVN with sell-up are also greater than those of all other O-D methods, even though less than those of EMSRb with sell-up.

This paragraph leads to the following very important conclusions:

- First of all, the sell-up heuristic, that is, accounting for sell-up in a fare class or virtual bucket environment, leads to substantial revenue gains (roughly 1%), whatever the network size. Accounting for sell-up is therefore a very robust and stable way to increase revenues for an airline using FCYM or GVN Revenue Management methods.
- Second, in a larger network GVN does much worse than it used to in the smaller network 6B. It is the Revenue Management method that suffers most from the change in network layouts.

6.2.5 Conclusion

This previous section on Network C allowed us to validate our network 6B results. Independently of what happens to the other O-D methods when we switch networks, the crucial observation to remember here is that the sell-up heuristic at differential input sell-up rates still performs very well and brings substantial revenue gains of about 1 percent at demand factor 1.0, whether we use EMSRb FCYM or GVN.

The network size therefore seems to have very little impact on the gains achieved by accounting for sell-up. On the other hand, the network size and layout did greatly modify the performance of all the O-D methods (compared to EMSRb with Booking Curve truncation and without sell-up). The most striking effect was that GVN actually had lower revenues than EMSRb when it did not account for sell-up. However, as soon as we account for sell-up in this Virtual Bucket environment, GVN recovers quickly, yet still has lower revenues than EMSRb with sell-up.

6.3 Observed Sell-up Rates (OSR)

Finally, in this last section we compare the sell-up rates we input to those we can evaluate with PODS. The output files in PODS give us each passenger's actual choice, given this passenger's first choice. While totally unavailable to the airlines in practice, the output of our simulation allows us, as we analyze the results of a case run, to evaluate the "true" sell-up rate, the Observed Sell-up Rate (OSR).

In the following paragraphs, we will first explain briefly what the output is and how we use it to compute the OSR. We will then compute these Observed Sell-up Rates for a few cases and compare them to our best-case estimates, namely our differential input sell-up rates. For the purpose of this analysis, we will focus on EMSRb, as we will discover that the OSR's are extremely different from our inputs

In the following paragraphs, we will be running Network 6B only, for consistency purposes. Indeed, the vast majority of our results were obtained in this smaller network and we feel it is more reasonable to keep using this network.

6.3.1 Computing the Observed Sell-up Rate

The PODS output files give us the actual choice of a passenger, given this passenger's first choice. Therefore, once we know the actual choice of the passenger given his first choice, we can count the number of passengers who chose a different Path/Fare combination, but on the **same** airline. Indeed, we implicitly define sell-up as the number of passengers who decided to fly on another Path/Fare combination, but on the same airline. Buying a higher fare on the competing airline does not constitute sell-up, but rather spill.

Moreover, given the layout of the network, and the fact that there is no favorite airline, we get two very important additional "constraints" imposed on passengers:

- All passengers who did not get their first choice can either sell-up, spill to the other airline or decide not to fly. They cannot sell-down.
- For the OSR computation, a passenger who sells up must do so on the same path as the network layout imposes that to fly from city A to city B, the passenger fly either airline A or B, each of them having exactly one departure per day.

Having set up this rather restricting picture, we now move on to the actual computation. This makes use of very simple mathematics: we first compute the observed sell-up rate by passenger type (pax) and fare class (c) using the following formula:

$$OSR_0(Pax, C) = \frac{\sum_{DIFF-F, SAME-A} PercentagePax}{1 - Percentage(SameFSameP)}$$

We sum the number of passengers who booked a different fare on the same airline (including those who booked a different path, if it were possible) and divide this number by the number of passengers who actually did not get their first choice and are therefore susceptible to sell up.

Once we have this intermediate Observed Sell-up Rate, we aggregate this number of all the passengers who flew in a given fare class, regardless of whether they are business or leisure passengers, for a given airline, in order to get a valid comparison with the sell-up rates we input in the **Belobaba-Weatherford** heuristic²². This is given by this next formula, which is a simple weighted average for all the passengers:

$$OSR(C) = \frac{BusPax \times OSR(Bus, C) + LeisPax \times OSR(Leis, C)}{BusPax + LeisPax}$$

For each fare class, we now have an Observed Sell-up Rate that we can compare to our estimate of the sell-up rate (input in the heuristic).

6.3.1.a Example

In the following paragraph, we give an example of how to compute the OSR for a given fare class and passenger type. The reader will recall that in Y class, there can be no sell up, as there is nowhere to sell up.

Table 33 shows the passengers' actual choices given their first choice, in terms of percentages. Given these percentages, we use our first formula to generate the last column of Table 33. In this column, we have the OSR by fare class, passenger type and airline. The reader will note here that the percentage of passengers who chose to fly on a different path and on the same airline (columns 7 and 9) is constantly 0. Indeed, Network 6B does not offer the possibility to choose amongst various paths on the same airline for a given Origin-Destination market.

Table 33: PODS output - Actual Choice Given First Choice Percentages

First Choice				Actual Choice								Observed Sell-up Rate
				SAME P SAME F	SAME P DIFF F	DIFF P SAME A	DIFF P SAME F	DIFF P DIFF A	DIFF P SAME A	DIFF P DIFF A	NOGO	
TYP	AL	CLS	#OBS									
1	1	1	4,065,216	94.9%	0.0%	0.0%	1.0%	0.0%	0.0%	0.0%	4.0%	
1	1	2	3,187,359	67.1%	16.3%	0.0%	1.0%	0.0%	0.0%	0.6%	15.0%	49.5%
1	1	3	483,239	50.1%	33.2%	0.0%	1.2%	0.0%	1.5%	14.0%		66.5%
1	1	4	164,037	31.7%	50.6%	0.0%	2.1%	0.0%	2.0%	13.5%		74.1%
1	2	1	3,932,481	95.0%	0.0%	0.0%	1.0%	0.0%	0.0%	4.0%		
1	2	2	3,085,545	67.0%	16.4%	0.0%	0.9%	0.0%	0.6%	15.1%		49.7%
1	2	3	469,093	50.3%	33.3%	0.0%	1.0%	0.0%	1.4%	13.9%		67.0%
1	2	4	159,277	33.4%	50.0%	0.0%	1.2%	0.0%	1.8%	13.6%		75.1%
2	1	1	15	100.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	
2	1	2	0	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
2	1	3	570	85.1%	1.9%	0.0%	2.1%	0.0%	0.4%	10.5%		12.8%
2	1	4	7,867,530	53.7%	5.1%	0.0%	3.6%	0.0%	0.3%	37.3%		11.0%
2	2	1	22	100.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	
2	2	2	0	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
2	2	3	567	84.8%	2.3%	0.0%	1.4%	0.0%	0.2%	11.3%		15.1%
2	2	4	7,685,930	56.3%	4.9%	0.0%	2.2%	0.0%	0.3%	36.3%		11.2%

Given these intermediate OSR's, we now aggregate the results over a fare class and we then obtain, for airline A:

Table 34: Observed Sell-up Rates for Airline A, by Fare Class

	OSR
B->Y	49.5%
M->B,Y	66.5%
Q->M,B,Y	12.9%

This simple example showed how to evaluate the Observed Sell-up Rates given the PODS output. We will be using these OSR's in the next paragraphs to compare them to our "optimal" differential input sell-up rates.

6.3.1.b Results

In the following paragraphs, we present the results of our simulations. Airlines A and B will both be using EMSRb with or without accounting for sell-up. In addition, we

will always be using Booking Curve detruncation. In the first paragraph, we focus on the Observed Sell-up Rates when neither airline accounts for sell-up. We will be looking for consistency in the OSR's as a function of the demand factor. Then, we will be looking at the influence of airline A accounting for sell-up on these OSR's. Finally we will conclude with a comparison of these rates with our "optimal" input sell-up rates.

6.3.2 Observed Sell-up Rates as a Function of the Demand Factor

In this paragraph, we present the result of our computation of the OSR's when neither airline accounts for sell-up, in a FCYM environment. The results of these simulations are shown in Table 35. We observe that these rates are consistent at all three demand factors. In the higher two classes, the sell-up rates seem to be decreasing when the demand factor increases from 0.8 to 1.2. On the other hand, the Observed Sell-up Rate for Q class barely increases with the demand factor. This can be explained by the fact that as demand increases, EMSRb will increase the protection for the higher classes, hence slightly reducing the Observed Sell-up Rates for these higher classes. At the same time, the lower classes will close down earlier, hence leading to an increase in the Observed Sell-up Rate and the NOGO rate (percentage of passengers who decide not to fly). The NOGO rate is not shown in this table, but it increases from about 24% to 54% for leisure passengers as the demand factor increases from 0.8 to 1.2.

Table 35: Observed Sell-up Rates as a Function of Demand Factor, EMSRb vs. EMSRb

Demand Factor		0.8	1.0	1.2
Sell-up Rate	B->Y	53.75%	50.00%	48.27%
	M->Y,B	70.85%	66.64%	65.28%
	Q->Y,B,M	12.61%	12.59%	13.08%

We also observe from Table 35 that the Observed Sell-up Rates are indeed differential, in that they are different for each fare class. However, they also are quite different from our input in several ways.

First of all, the values of these rates are quite different from what we input, and generally much higher, except in Q class, where the rates are much lower.

Second, these rates are not spread out in the same way. Our choice of sell-up rates was based on the assumption that the higher the fare class, the greater the willingness-to-pay, and therefore the higher the probability that a passenger in that class will accept to sell-up. However, this is not what we observe here.

In summary, this section suggests that the Observed Sell-up Rates are rather stable as a function of the demand factor, yet are quite different from what we input. As far as the difference between the OSR and the input sell-up rates is concerned, there are two possible explanations. First of all, it could be that the method we use to compute

the OSR is not valid or that the bookings reported by PODS are biased by some unknown factor. However, this is unlikely. The more probable explanation is that the EMSRb sell-up heuristic is not very realistic.

6.3.3 Influence of Accounting for Sell-up on Observed Sell-up Rates

In the following paragraphs, we run the simulator with airline A and/or airline B accounting for sell-up either at constant input sell-up rates or at differential input rates. We focus on the influence of these changes to the airlines' Revenue Management methods on the Observed Sell-up Rates.

Table 36 summarizes our results, for airline A, in terms of Observed Sell-up Rates.

Table 36: OSRs as a Function of the Input Sell-up Rate, DF 1.0

DF 1.0	Input Sell-up Rates				
	Base	20%-0%	20%-20%	DS-0%	DS-DS
B->Y	50.0%	54.3%	53.8%	53.7%	54.8%
M->Y,B	66.6%	70.8%	69.9%	66.6%	68.3%
Q->Y,B,M	12.6%	13.2%	13.8%	10.8%	12.1%

It appears from Table 36 that accounting for sell-up, be it for one airline or both tends to lead to an increase in the Observed Sell-up Rate in classes B and M. In class Q, however, the Observed Sell-up Rate seems to remain more or less constant regardless of whether or not one or both airlines account for sell-up. However, a general conclusion from Table 36 is that the Observed Sell-up Rates tend to remain very stable as we change the input sell-up rates in the **Belobaba-Weatherford** heuristic²².

In summary, we observe from the above results that the Observed Sell-up Rates seem to not depend very much upon whether or not the airlines account for sell-up, in the FCYM case.

6.3.4 Summary

The first major observation from this section is that the Observed Sell-up Rates, as computed through the output of PODS, are very stable, regardless of the demand factor or the input sell-up rate if one or both airlines are accounting for sell-up.

The second critical observation is related to the values of these Observed Sell-up Rates. First of all, the Observed Sell-up Rates are indeed differential, as suggested by **Bohutinsky**²⁰ and confirmed by our results in this chapter. However, they are not in the same "order" as our input sell-up rates. Indeed, our assumption was that the sell-up rate increased with the fare class. We believed, as reflected by our choice of sell-up rates, that as passengers by a higher fare, they have higher willingness-to-pay, and therefore a greater probability to sell up to a higher class. However, the Observed Sell-up Rates are high in B class, highest in M class and lowest in Q class. Second, these sell-up rates are much higher than our input sell-up rates in B and M

classes, and lower in Q class. The reader will recall here that our differential sell-up rates were 45%, 30% and 20% from top down, while the Observed Sell-up Rates are in the neighborhood of 50%, 67% and 13%.

Our final observation in this section is that these sell-up rates cannot be used as such in the heuristic. Indeed, as we explained in 4.1.1.a, too high sell-up rates lead to overprotection and then revenue losses when the heuristic explodes. When we input these sell-up rates into the heuristic, we ended up with revenue losses of about 10% to 17% depending on the demand factor. These differences in Observed and input Sell-up rates may be explained by the fact that the forecaster implicitly estimates part of the sell-up by forecasting demand for a given class regardless of whether it is direct demand or demand that is the results of a passenger selling up. The major reason for this being that the forecasts are based on historical data and that historical data includes the sell-up of some passengers towards higher classes.

6.4 Conclusion

This chapter allowed us to draw the following major conclusions:

- First of all, accounting for sell-up allows revenue gains of about 1 percent with FCYM at demand factor 1.0. These revenue gains are lower than those of most of the other Revenue Management methods available to us in PODS. However, they compare to the revenue gains of Netbid, in Network 6B. The revenue gains of GVN with sell-up bring tremendous improvement and are comparable to the “best” O-D methods available (ProBP and DAVN) in this “virtual-friendly” network.
- Second of all, the revenue gains achieved by accounting for sell-up seem to hold very well in a larger network. In particular, as all the other O-D methods suffer from the new network layout, EMSRb with sell-up performs extremely well and becomes the best method. GVN on the other hand also suffers dramatically from the switch to Network C, yet its losses are turned into gains when accounting for sell-up.
- Third and finally, the “optimal” sell-up rates, as defined in Part II of this thesis, seem to be very different from those we found to be observed in PODS. However, as we explained at the end of the previous section, this may be related to the performance of the forecaster.

CONCLUSION

Summary of Findings

The reader will recall that our primary objective in this thesis was to estimate the influence of sell-up models on the revenues of a leg-based Revenue Management algorithm. In addition, we also wished to determine whether it would be possible for airlines to achieve revenue gains comparable to those of O-D Revenue Management methods, compared to EMSRb, while remaining in a Leg-based Revenue Management environment. Our goal was to provide insight into whether a small airline may be able to increase its revenues by accounting for sell-up and improving its forecasting capabilities, in its less advanced Leg-based Revenue Management system, and forgo the cost of installing a much more expensive O-D system.

Our results showed that it is possible to increase revenues for an airline using Leg-based Revenue Management algorithms whether it uses EMSRb or GVN. More specifically, we achieved revenue gains ranging from 0.7% to 2.8% at demand factor 1.0 depending on the competitive situation and the Revenue Management algorithm used by the airline, as discussed in this thesis. The fundamental results that emerged from the thesis are summarized in the following paragraphs.

First of all, inputting a sell-up rate in the **Belobaba-Weatherford** heuristic²² and using this heuristic in the Revenue Management algorithms (EMSRb or GVN) can generate revenue gains for the airline implementing this change. However, this happens only in the case when the sell-up rates are properly chosen by the user. As we discussed in Part I, Chapter 3.3, the chosen sell-up rates should match, as best we can estimate, the propensity of a certain category of passengers to sell up. As we discussed earlier in the thesis, it is not the purpose of this thesis to quantify the sensitivity of the revenue gains to the choice of the sell-up rates. We focus on the range of the revenue gains we can achieve by accounting for sell-up in a leg-based environment.

Second, the best sell-up rates, as suggested by **Bohutinsky**²⁰, are differential. We showed in the thesis that these rates depend on the Revenue Management algorithm used (EMSRb or GVN) but are always designed to match passengers' propensity to sell up. Indeed, different Revenue Management algorithms require different inputs in terms of sell-up rates, depending on whether they use a set of virtual buckets or do the computations on a fare class basis. EMSRb and GVN, for example, do not use

the same mapping of the fares, and therefore, we had to define two different types of differential input sell-up rates. In all cases, assuming differential sell-up rates brought higher revenue gains.

Third, we established that, whether only one airline, or both airlines, account for sell-up, results in a change in the revenue gains observed by airline A: Airline A's revenue gains are lower when the competitor accounts for sell-up compared to when the competitor did not account for sell-up. However, the fundamental conclusion from our results is that both airlines still benefit from accounting for sell-up. Another way to explain this is that airline Revenue Management is not a zero-sum game: If both airlines improve their Revenue Management algorithms (by accounting for sell-up, for example), they can both benefit and improve their revenues, overall increasing the total network revenues.

Fourth, in terms of the effect of the detruncation method, unlike the results shown by **Zickus**⁹, switching to another of the three available detruncation methods (from Booking Curve detruncation) does not lead to a substantial additional increase in revenues over the case when we accounted for sell-up. Indeed, we showed that both detruncation and the sell-up heuristic had the same effect: increase the protection levels in the higher classes. However, accounting for sell-up leads to more sensitive variations in the protection levels. The sell-up heuristic allows specification of the sell-up rate for each fare class or virtual bucket and, this, in turn impacts differently on the additional protection for each fare class (or virtual bucket). On the other hand, the detruncation method has more general effect that cannot be adapted to each fare class or virtual bucket. Overall, combining the sell-up heuristic with more aggressive detruncation is excessive. We also point out here that our results do not contradict **Zickus's** results in any way. Accounting for sell-up and switching to another detruncation method have similar impacts on the Revenue Management algorithms, yet accounting for sell-up leads to higher gains. Moreover, accounting for sell-up and switching to more aggressive detruncation have overlapping effects that eventually lead to overprotection.

Finally, in terms of the network size, we also showed that all the results observed on the small network (6B) also apply to the bigger network (C) environment. In particular, the sell-up heuristic performs very well with GVN or EMSRb in that it is responsible for revenue increases with both these Revenue Management algorithms (in Network 6B or Network C). The revenue gains are in the neighborhood of 1%. This final comment is very important in that it allowed us to confirm that our conclusions on the effect of sell-up models on revenues does not depend on the network size or the demand distribution in terms of the percentage revenue gains that can be obtained.

Figure 48: Revenue Gains Due to the Incorporation of Sell-up Models

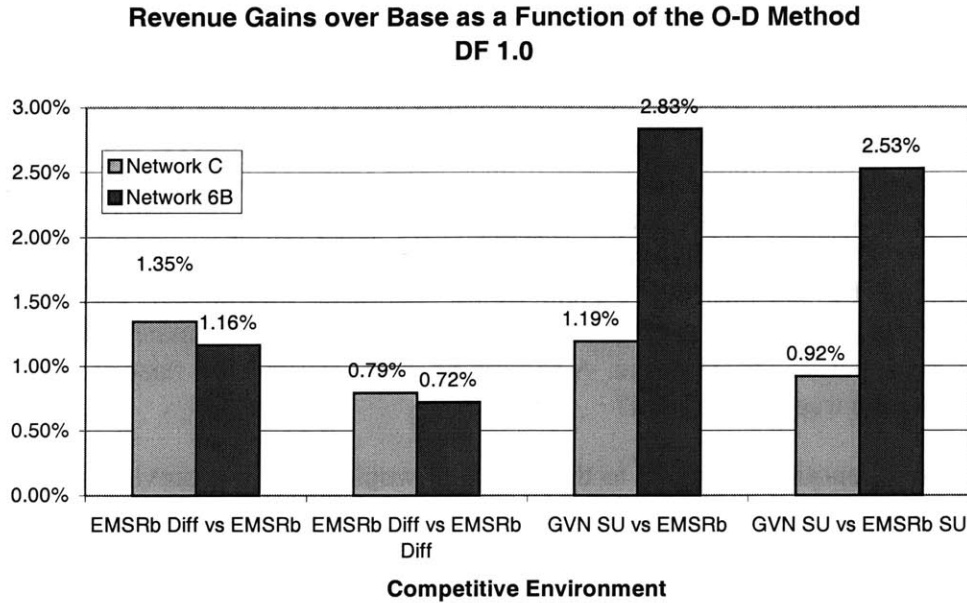


Figure 48 summarizes the revenue gains attributable to the incorporation of the sell-up models in EMSRb or GVN. This chart shows the difference in gains in the two networks tested. The difference in revenue gains for GVN can be explained by the difference in load factors between Network 6B and Network C.

Overall, Figure 48 shows that there is a non-negligible revenue gain to be achieved by accounting for sell-up, be it in the EMSRb or the GVN Revenue Management environment. Even in the worst-case scenario for GVN, there still is about 0.7% gains to be obtained from accounting for sell-up, over the base case scenario (EMSRb vs. EMSRb). This confirms our initial hypothesis, which was that there were gains to be achieved from switching to Leg-based models that account for sell-up without necessarily implementing more complex O-D algorithms.

Figure 48 summarizes most of our results in terms the impact of sell-up on airline revenues. It shows that accounting for sell-up can lead to non-negligible revenue gains in both our networks (6B and C). It also shows that, even though the network layout and distribution of demand clearly impacts the gains, accounting for sell-up invariably leads to revenue gains of about 1 percent.

Future Research Directions

Two major future research directions will be discussed in the following paragraphs:

- First of all the possibility of an algorithm that would estimate the actual sell-up rate and use it as an input to the heuristic.

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- Second of all, the incorporation of a sell-up heuristic in other O-D methods that do not rely on leg-based algorithms to determine the protection levels.

Estimating the Observed Sell-up Rates

In our simulations, we used the PODS output files to evaluate the Observed Sell-up Rates, based on true data. However, it is clear that the airlines do not have this type of information available. Indeed, passengers are not tested for their willingness to sell-up. It could be possible to record the number of passengers who accepted to pay a higher fare given that their first choice was unavailable. However, again as discussed by **Bohutinsky**²⁰, this would be both impractical and difficult to implement. Moreover, even in our experiments, we only estimate the sell-up rates that we input in the heuristic. A legitimate question that we raise is: How do we evaluate the true sell-up rates?

Therefore, another approach to this problem would be to use previous booking data to forecast the sell-up rates. This would also potentially eliminate the overprotection that we expect to observe with the static sell-up rates. Indeed, as we start accounting for sell-up through the heuristic, we increase the protection for the higher fare classes or buckets. Therefore, the recorded demand should increase in these fare classes thus increasing the forecasts and in turn the protection levels in these higher fare classes. If the input sell-up rates are not modified, the airline should ultimately end up in a situation where it is overprotecting in the higher fare classes. Such a forecaster has been developed by **Hopperstad** and briefly tested. The first results show that the forecast sell-up rates are inconsistent with both the observed and the input sell-up rates. We do not present any results in this thesis regarding the estimator, as represents a preliminary effort that requires further research.

This estimator should, in the long run, allow the airlines to increase their revenues by accounting for sell-up in a way that responds to demand and therefore is not based on educated guesses, as was shown in this thesis.

Incorporating Sell-up Models in other O-D Methods

In this thesis, we only tested the effect of the **Belobaba-Weatherford** heuristic²² on the leg-based EMSRb and GVN Revenue Management methods. Indeed, the heuristic was designed to work with the EMSR model. As we showed in the last part of the thesis, the revenue gains achieved by these two methods, when the airline accounted for sell-up, were substantial and even above those of all the other O-D methods, including ProBP, when they did not account for the possibility of sell-up.

A legitimate future direction then is to explore the possibility to include a sell-up model in these other O-D methods. DAVN also uses the EMSR algorithm to compute the protection levels. Therefore, it is possible (and this option is offered in PODS) to use the **Belobaba-Weatherford** heuristic²² with DAVN. However, the LP bid price methods do not yet offer the possibility to account for sell-up. Therefore, future directions include developing a heuristic that would enable us to account for

sell-up in the bid price environments on the one hand, and testing the heuristic on these methods to see how much revenue gains it can bring to the airline using it, on the other hand.

GLOSSARY

Detruncation

In PODS, detruncation is the process of estimating the actual demand for a flight. Indeed, flights are limited in capacity and therefore a full flight could have had a higher demand than capacity allowed.

Forecasting

Forecasting consists in estimating future demand for a given flight, based on previously recorded demand only. In a global setting, it encompasses two concepts, detruncation and forecasting. Detruncation is explained above. The second concept, forecasting, is the action of estimating future demand based on previously estimated demand (through the detruncator).

ODF

Origin-Destination Fare. This term refers to the demand on a given origin-destination market at a specific fare.

PODS

Passenger Origin Destination Simulator. This simulator was developed by Hopperstad, Berge and Filipowski at the Boeing Company. It allows the user to simulate a competitive network of airlines.

Revenue Management

Revenue Management, also known as Yield Management, is the process of setting protections levels and booking limits on each fare class in order to maximize the revenues of the airline.

Sell-up

Sell-up is the human behavior that lets passengers travel at a higher fare than the one initially requested. This is due to the fact that each passenger has a maximum willingness-to-pay, and is therefore willing to spend as much money as this price.

When offered a fare, the passenger will accept it if it is below his (her) maximum willingness-to-pay. In the thesis we distinguish different types of sell-up.

Assumed Sell-up - Input Sell-up

This is the sell-up that we use as a part of the **Belobaba-Weatherford** heuristic. We estimate the sell-up rates based on previous simulations and previous work by **Bohutinsky**. This sell-up is no more than an educated guess.

Observed Sell-up

This sell-up is based on PODS outputs. The output files give us the actual choice of every passenger given their first choice. Based on this data, we compute an estimate of the “true” sell-up rates (c.f. Part II. Chapter 6.3.1).